McKay Correspondence between Physical World and Mental World

Frank Dodd (Tony) Smith, Jr. - **UPDATE December 2018**

Saul-Paul Sirag, in Jeffrey Mishlove's 1993 book "Roots of Consciousness"
and in his 2016 book ADEX theory, describes the relationship between
the Physical World and the Mental World in terms of
in terms of the McKay correspondence between subgroups of SU(2) and Lie Algebras.
My 2010 paper that follows this 2018 Update was based on his approach of identifying
the Physical World with SU(2) subgroups and the Mental World with Lie Algebras.
This update page is based on my present view of Cl(16) Physics, identifying
the Mental World with SU(2) subgroups and the Physical World with Lie Algebras.

Therefore, my updated understanding interchanges the terms Body and Mind so take that into account as you read the 2010 paper that has not been updated by editing.

560 TriVectors of Cl(16) = 10 copies of 56-dim Fr3(O) Freudenthal Algebra
which is complexification of 27-dim J3(O) Jordan Algebra with F4 symmetry
whose traceless part is 26-dim J3(O) of 26D String=World-Line Theory
whose spin-2 part gives Bohm Quantum Potential and **Quantum Consciousness.**
F4 has Root Vectors = 24-cell + dual 24-cell. 24-cell = 8 Octahedra.
Octahedron is Jitterbiug transform of Cuboctahedron and Icosahedron.
Icosahedral Double Group ID has 120 elements and 8-dim Catastrophe space and represents Universal Mind.
Each Universal Mind Cell of the 26D Lattice has 2^16 = 65,536 elements.

16 Vectors of Cl(16) = 16 dimensions of Lie Ball Complex Domain with symmetry D5 / D4 x U(1) and Lie Sphere Shilov Boundary Spacetime RP1 x S7

120 BiVectors + 128 half-Spinors of Cl(16) give E8 and a Physics Lagrangian
that describes Elementary Particles and Schwinger Sources and Nuclei and Atoms
that make up Microtubules. **E8 has 248 elements and 8-dim Root Vector space and represents Universal Body.**
Maximal 40-micron Microtubule has 65,000 Tubulin Dimers.

8-dim ID Catastrophe MInd Space = 8-dim E8 Root Vector Body Space

Icosahedral Double Group Universal Mind and E8 Universal Body are in McKay Correspondence with each other so there is Universal Mind Cell - Universal Body Microtubule Resonant Connection.

The Universal Geometric Entity is the Real Clifford Algebra Cl(16) which contains both Universal Body and Universal Mind and the 16-dim Lie Ball with 8-dim Lie Sphere Spacetime.

McKay Correspondence between Physical World and Mental World

Frank Dodd (Tony) Smith, Jr. - around Halloween 2010

Saul-Paul Sirag, in an Appendix to Jeffrey Mishlove's book "Roots of Consciousness" (Council Oak Books 1993), describes
the relationship between the Physical World and the Mental World
in terms of
the correspondence between McKay Subgroups of SU(2) and Lie Algebras
using
the complex 133-dimensional exceptional Lie algebra E7 as fundamental.
This paper is an attempt to follow Saul-Paul Sirag's approach except for one difference: using real 248-dimensional exceptional Lie algebra E8 as fundamental.
The basic idea is set out by Saul-Paul Sirag in the Appendix:
"...[this quote is modified by using E8 terminology]...
There is a Universal Body [120-dim ID group with 8-dim catastrophe perturbation space]...
There is a Universal Mind [Lie algebra E8 generated by reflections in R8]...
The intersection of the two
...[8-dim R8 as ID Catastrophe Control Pertubation Space
and
8-dim R8 as E8 reflection space and]...
is Universal Consciousness
...
There is a Universal Geometric Entity ...[UGE]... which is the direct product of ... the Universal Body ... and ... the Universal Mind
...
Notice that from the point of view of ...[UGE]... the system is monistic,
but from the point of view of the constituent algebras ...[ID and E8]... the system is dualistic. ...".

Table of Contents:

E8 Lie Algebra Balance Numbers and Weyl Group

John McKay said on usenet sci.math in 1993:
"... For each finite subgroup of SU2, we get an affine Dynkin diagram ...

```
                         3
                         |
E[8]        1-2-3-4-5-6-4-2
```

...
The [McKay] correspondence is ...
E[8] ...[corresponds to] 2.Alt[5] = SL(2,5) binary icosahedral [ID group]
...
There are [8+1 = 9 **balance numbers** for E8]...
The sum of the numbers [1+2+3+4+5+6+4+2+3 = 30 is] h = Coxeter number.
The sum of the squares is the order of ...[120-element ID for E8]...
The numbers are the degrees of irreducible representations of G.
They are first Chern numbers (singularities).
They are the periods of products of pairs
of Fischer involutions mod centre ... E[8] ...[for]... Monster
...
for the E8 - icosahedral ... case, the singularity is x^2+y^3+z^5=0 ...".

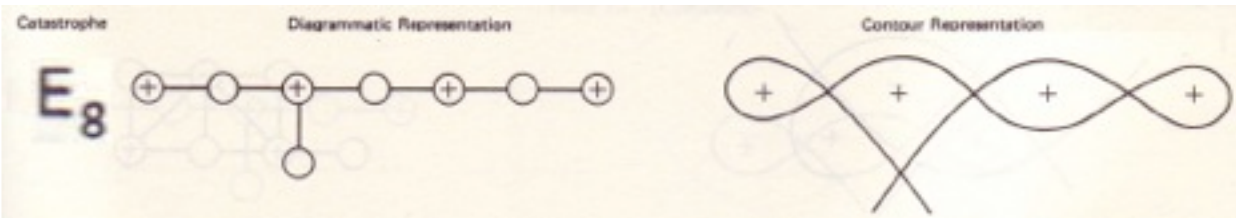
In a 1993 email John McKay said:
"... The connection with the monster is not understood at all,
however it has as consequence that each node on the extended
diagrams is attached to a modular function ...".]

Saul-Paul Sirag, in the Appendix,
noted that the numer of mirrors in E8 reflection space is 8 h / 2 = 120.
He also noted that the product of the E8 balance numbers is 128 x 27 x 5
which when multiplied by 8! = 8x7x6x5x4x3x2x1 gives
the order 696,729,600 of the Weyl reflection group of E8 in R8 Euclidean 8-dimensional space.

Saul-Paul Sirag, in the Appendix, used a correspondence between E7 balance numbers and degrees of catastrophe terms that is not a general correspondence and in particular breaks down for E8, as Peter Slodowy said in a 1993 email message to John McKay.
Since that is not a valid correspondence in general and not valid for E8 in particular, that correspondence will not be discussed further in this paper.

Icosahedral Double Group Catastrophe Control Space

Robert Gilmore, in his book "Catastrophe Theory" (Dover 1981) said:
"...[The Icosahedral Double Group Catastrophe]... E8 ...[has]...
Catastrophe Germ ... X^3 + Y^5
...[with]... Perturbation ...
a1 Y + a2 Y^2 + a3 Y^3 + a4 X + a5 X Y + a6 X Y^2 + a7 X Y^3
...



...

The germs E_6, E_8 are

$$E_6: \quad f(x, y) = x^3 + y^4$$

$$E_8: \quad f(x, y) = x^3 + y^5$$

The rules for determinacy and unfolding are particularly easy to carry out for E_6 and E_8 because both $\partial f/\partial x$ and $\partial f/\partial y$ are monomials. These calculations are summarized diagrammatically in Fig. 23.4.

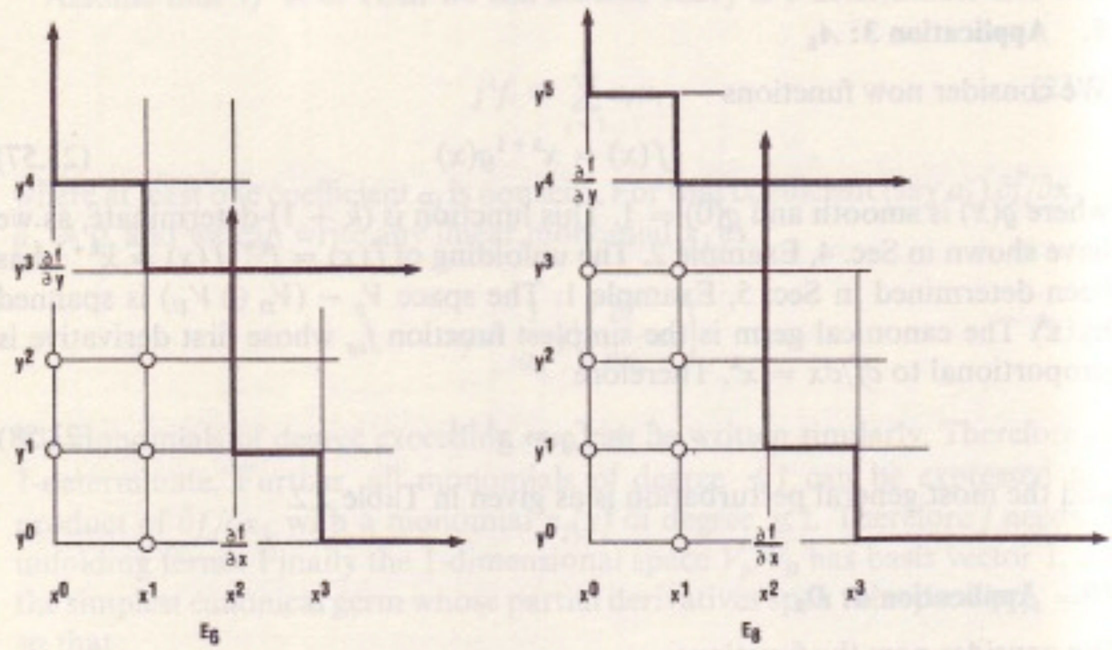


Figure 23.4 For E_6 and E_8 all monomials of degree 4 and 5 can be expressed in the form $(\partial f/\partial x_i)m_j$. The unfolding terms are represented by open circles. We exclude the constant term.

...
for E8 ...[with]... control parameter space R7 ...[basis {a1,a2,a3,a4,a5,a6,a7}]... the maximum number ... of isolated critical points ..[is]... 8 ...".

In the Appendix, Saul-Paul Sirag did not "exclude the constant term" as Robert Gilmore did, so, if we follow Saul-Paul Sirag's approach and effectively add a control parameter a0, we see that the ID E8 Catastrophe Control Parameter Space is R8 with basis {a0,a1,a2,a3,a4,a5,a6,a7}
and
the Catastrophe Control Parameter Polynomials of ID are invariant under the Weyl Group of the E8 Lie Algebra Root Vector Reflections.

Since the E8 Weyl Group generates the E8 Lie Algebra from the Root Vector Reflections in R8,
and
also generates the Catastrophe Control Parameter Polynomials of ID

it is clear that R8 Reflections are the common ground from which
the ID Universal Body and the E8 Universal Mind both emerge,
so
it is natural that
the R8 Reflection Space be identified as the Universal Consciousness.

R8 Universal Consciousness and World Line Strings

R8 Reflection Space of Universal Consciousness combines
Root Vector Reflections of the E8 Universal Mind
with
Weyl Group Catastrophe Control Polynomials of the ID Univeral Body.

A way to see this interaction in greater detail is to expand R8 to 10 dimensions and then formulate a Bohmian Quantum Potential in terms of a 26-dimensional Bosonic String Theory that gives a superposition/separation (see http://tony5m17h.net/Rzeta.html#supsepBSgrav) gravity-type potential needed for Penrose/Hameroff Quantum Consciousness.

In terms of the E8 sector, the expansion from 8 to 10 dimensions
starts with 8-dim as sum of 4-dim Minkowski plus 4-dim CP2 Internal Symmetry
and then expands the 4-dim Minkowski to 6-dim Conformal Spin(2,4) spacetime
resulting in 6+4 = 10 dimensions.

In terms of the Icosahedral Double sector, the expansion from 8 to 10 dimensions
starts with Catastrophe Control Parameter space (basis {a0,a1,a2,a3,a4,a5,a6,a7})
and
adds two basis elements {X,Y} of ID Catastrophe Germ space whose polynomials
are invariant under the Icosahedral Double Group ID
resulting in 8+2 = 10 dimensions.

The strings in this version of Bosonic String Theory correspond to Possible World-Lines in the Many-Worlds and there is no naive 1-1 supersymmetry.
(see http://tony5m17h.net/E6StringBraneStdModelAR.pdf)

ID Universal Body and Standard Model unified with Gravity

We can construct a realistic physics model,
including the Standard Model unified with Gravity,
from the 600cell with 120 vertices,
which 120 vertices correspond to the 120 elements of ID.

(see http://tony5m17h.net/TetraJJDECF.pdf)

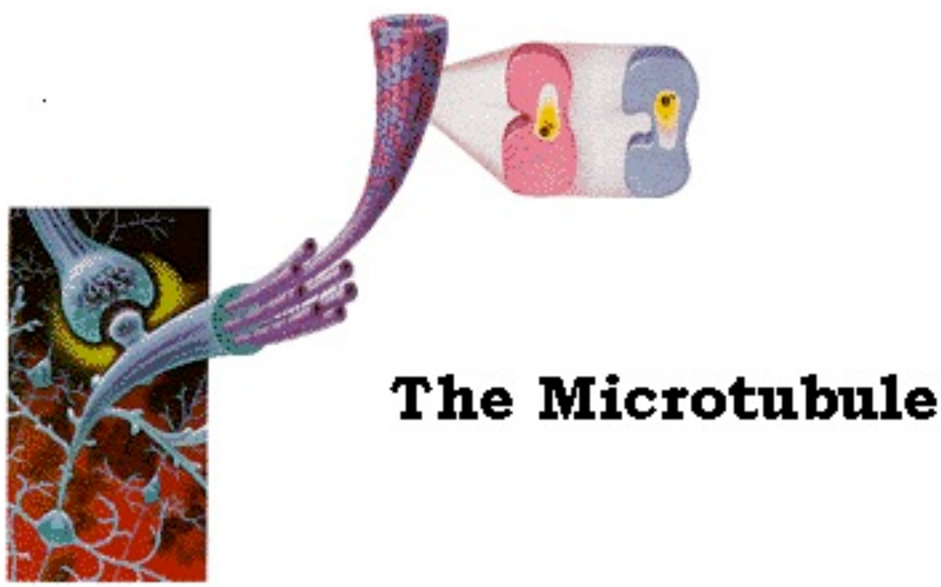
E8 Universal Mind and Clifford Algebra

E8 is part of the Cl(16) Clifford algebra (bivector plus halfspinor).
Clifford algebras describe quantum computers
which in turn describe consciousness.
Any Clifford algebra no matter how large
can be described in terms of Cl(16) factors
due to Cl(8) periodicity of real Clifford algebras.

For Quantum Consciousness details see
http://tony5m17h.net/AfricaCl8x8E8.pdf
http://tony5m17h.net/E8CCTS12a.pdf
http://www.valdostamuseum.org/hamsmith/QuanCon.html
http://tony5m17h.net/QuantumMind2003.html

Here is a rough outline of Clifford Algebra Quantum Consciousness:

The human brain contains about 10^18 tubulins in cylindrical microtubules.
Each tubulin contains a Dimer that can be in one of two binary states.



in the illustration (from a Rhett Savage web page),
the red dimer has its electron in the down state
and the blue dimer has its electron in the up state.
Eac tubulin is about 8 x 4 x 4 nanometers in size
and contains about 450 molecules (amino acids) each with about 20 atoms.

If about 10% of the brain is involved in a given conscious thought,
then, since $2^{56} = 7.2 \times 10^{16}$,
it involves about 10% of 10^{18} or about $2^{56} = 2^{(8x7)}$ tubulins,
so the mathematics of that thought is described by the Clifford algebra Cl(56)
which is (by 8-periodicity) Cl(56) = Cl(8x7) =
= Cl(8) x ...(7 times tensor product)... x Cl(8) =
= 7 states of the basic Clifford algebra Cl(8)

That may account for "The Magical Number Seven, Plus or Minus Two:
Some Limits on our Capacity for Processing Information"
by George Miller available on the web at http://psychclassics.yorku.ca/Miller/
where he refers to a

"... basic process that limits our unidimensional judgements to about seven categories ...".

He distinguishes "the span of absolute judgment" with limit of about seven
from "the span of immediate memory" as to which Wikipedia says
the limit "... is probably around three or four".

Since $2^{(8x4)} = 2^{32} = 4.3 \times 10^{9}$ and $2^{(8x3)} = 2^{24} = 1.7 \times 10^{7}$
it may be that the span of immediate memory corresponds to
the square root of the number of tubulins $\mathrm{sqrt}(10^{18}) = 10^{9}$
because
if the tubulins required for a full judgment thought process
were to be represented as an area

then the tubulins required for immediate memory (lasting only a short time and allowing most (almost all) brain capacity to be used for other things) might be represented by a line (square root of the area)

As to how the thought formation process works, my model (which is of the Penrose-Hameroff type) indicates that all 2^{56} of the tubulins are coherently in phase together, forming a coherent quantum state containing all possible thought outcomes (i.e., all Bohm possibilities or all possible Worlds of the Many-Worlds).

After a time the coherent state dechoheres into a single outcome state
that is the thought that is the result of the process (Penrose calls that Orch OR,
or "Orchestrated Objective Reduction of Quantum Coherence".)

Penrose proposes that Orch OR is caused by Quantum Gravity, occurrring
after expiration of the time allowed for that many tubulin states to be held
in a coherent superposition.

The time at which decoherence takes place and a thought is formed
can be calculated using quantum gravity ideas,
and
the calculation results are consistent with the data of the human brain.
For details see http://tony5m17h.net/QuanCon.html

The formation of a human thought by 2^56 tubulins
is similar to the formation of our Universe beginning with an initial single
Quantum Fluctuation Big Bang corresponding to
the Clifford Algebra Cl(0) with 2^0 = 1 element,
then non-unitary Octonionic inflation proceeding to

Cl(1) with 2^1 = 2 elements
Cl(2) with 2^2 = 4 elements
Cl(4) with 2^4 = 16 elements
Cl(8) with 2^8 = 256 elements
Cl(16) = Cl(8) x Cl(8) with 2^16 = 65,536 elements
Cl(32) = Cl(8) x ...(4 times tensor product)... x Cl(8) with 2^32 = 4.3 x 10^9
Cl(64) = Cl(8) x ...(8 times tensor product)... x Cl(8) with 2^64 = 1.8 x 10^19

at which level octonion self-reflexivity of Cl(64) = Cl(8x8) causes decoherence
and the beginning of M4xCP2 Kaluza-Klein quaternionic unitary expansion
and the end of inflation as described by Paula Zizzi
(see http://en.wikipedia.org/wiki/Paola_Zizzi
and http://www.valdostamuseum.org/hamsmith/cosm.html#instexp).

The model is consistent with the Big Bang and Inflation
being a Thought in the Mind of God

Paula Zizzi wrote a paper "Poetry of a Logical Truth" in which she said
"... We believe that the sense of the “Self” in human minds arises when
a quantum mental state undergoes a mirror measurement.
...
“mirror measurement” would correspond to a “mirroring” of the “Self",
and some anomaly in that process might be responsible for a mental disease like
the “borderline”.
...
The communication channel between a mind in a classical state and another mind
in a quantum state, corresponding to a (external) quantum measurement,
in the framework of standard quantum logic, can be viewed as the old kind of
therapy.
... A new kind of therapy might require the therapist to reconstruct the
paraconsistent logic of the patient from the data ...”.

Her approach seems to be a promising way to use quantum consciousness
for therapy for some hard-to-treat psychiatric disorders.

Universal Geometric Entity

The Universal Geometric Entity (UGE)is the direct product of

the Universal Body (modelled from 120-dimensional ID)

and

the Universal Mind (modelled from 248-dimensional E8
which in turn is constructed from 256x256-dimensional Cl(16)).

Appendices: ID Character Table and E8 Representations

David Ford and John McKay wrote in the book "The Geometric Vein" (Springer-Verlag 1981):
"... The columns of the character tables of ... the binary icosahedral group ...[ID Icosahedral Double Group]... of order 120
are
the (suitably normalized) eigenvectors of the Cartan matrices of type ... E8 ...

[Let gr = (1/2) (- 1 - sqrt(5)) and GR = (1/2) (- 1 + sqrt(5)) and note that gr + GR = -1]
...

1	1	1	1	1	1	1	1	1
2	-2	0	-1	1	GR	gr	-gr	-GR
2	-2	0	-1	1	gr	GR	-GR	-gr
3	3	-1	0	0	-gr	-GR	-GR	-gr
3	3	-1	0	0	-GR	-gr	-gr	-GR
4	4	0	1	1	-1	-1	-1	-1
4	-4	0	1	-1	-1	-1	1	1
5	5	1	-1	-1	0	0	0	0
6	-6	0	0	0	1	1	-1	-1

...".

The E8 Coxeter-Dynkin diagram, in which
each vertex corresponds to a fundamental representation, is

```
                                               147250
                                                 |
248 - 30380 - 2450240 - 146325270 - 6899079264 - 6696000 - 3875
```

E8 has 8 balance numbers which are 1,2,3,4,5,6,4,2,3
Their product is 128 x 27 x 5 which multiplied by 8x7x6x5x4x3x2 gives
the order 696,729,600 of the Weyl reflection group of E8

A Cartan matrix for E8 is

2	0	-1	0	0	0	0	0
0	2	0	-1	0	0	0	0
-1	0	2	-1	0	0	0	0
0	-1	-1	2	-1	0	0	0
0	0	0	-1	2	-1	0	0
0	0	0	0	-1	2	-1	0
0	0	0	0	0	-1	2	-1
0	0	0	0	0	0	-1	2

Note that E8 can be constructed from the representations of E6 and D8.

The grade-1 vector representation of D8 is 120-dimensional.
The half-spinor representation of D8 is 128-dimensional.

The adjoint representation of E8 is 120 + 128 = 248-dimensional.

D8 and E6 both have trivial 1-dimensional scalar representations.

E6 has 27-dimensional and 78-dimensional representations.

The construction of E8 Dynkin representations

```
                                                   147,250
                                                      |
248 - 30,380 - 2,450,240 - 146,325,270 - 6,899,079,264 - 6,696,000 - 3,875
```

is as follows:

First, construct the exterior/wedge products of the E8 adjoint 248:

The grade-1 part has dimension 248.

The grade-2 part has dimension 248/\248 = 30,628.

The grade-3 part has dimension 248/\248/\248 = 2,511,496.

The grade-4 part has dimension 248/\248/\248/\248 = 153,829,130.

The grade-5 part has dimension 248/\248/\248/\248/\248 = 7,506,861,544.

Now:

Keep the grade-1 part of dimension 248.

Subtract off 248
from 248/\248 = 30,628 to get 30,380.

Subtract off 2 x 248/\248 = 2 x 30,628
from 248/\248/\248 = 2,511,496 to get 2,450,240.

Subtract off 2 x 2,511,496 and 2,450,240 and 30,628
from 248/\248/\248/\248 = 153,829,130 to get 146,325,270.

Subtract off 2 x 153,829,130 and 2 x 146,325,270
and 2 x 2,511,496 and 2,450,240 and 248
from 248/\248/\248/\248/\248 = 7,506,861,544 to get 6,899,079,264.

These are 5 of the 8 fundamental representations of E8.

They, like the D(N) and A(N) series constructions,
are all in the same exterior algebra (of /\248),
and so can be represented as the vertices of a pentagon.

What about the 6th and 7th fundamental representations of E8?

Consider the 27-dimensional E6 representation space.
Add 32 copies of the 128-dimensional D8 half-spinor space,
and subtract off
one copy of the 248-dimensional E8 representation space
to get
a 27 + 32 x 128 - 248 = 3,875-dimensional representation space.

Now, consider the antisymmetric exterior wedge algebra
of that 3,875-dimensional space.

The grade-1 part has dimension 3,875.
The grade-2 part has dimension 3,875/\3,875 = 7,505,875.

Now:

Keep the grade-1 part of dimension 3,875.

Subtract off 5 x 147,250 and 2 x 30,628
and 3 x 3,875 and 3 x 248
from 3,875/\3,875 = 7,505,875 to get 6,696,000.

They are the 6th and 7th fundamental representations of E8.
Since they are not in the same /\248 exterior algebra as
the 5 pentagon-vertex fundamental representations of E8,
they should not be vertices in the same plane as the pentagon.
However, since they are in the same /\3,875 exterior algebra,
they should be collinear, one above and one below the pentagon,
thus forming a pentagonal bipyramid.

What about the 8th fundamental representations of E8?

Consider 2 x 24x24 - 1 = 2 x 576 - 1 = 1,151 copies of
the 128-dimensional D8 half-spinor space,
and subtract off
one copy of the 78-dimensional E6 representation space
to get a representation space of dimension
1,151 x 128 - 78 = 147,328 - 78 = 147,250.

Now, consider the antisymmetric exterior wedge algebra
of that 147,250-dimensional space.
The grade-1 part has dimension 147,250.

It is the 8th fundamental representation of E8.
Since it is not in the same /\248 exterior algebra as
the 5 pentagon-vertex fundamental representations of E8,
it should not be a vertex in the same plane as the pentagon.
Also, since it is not in the same /\3,875 exterior algebra as
the two bipyramid-peak-vertex fundamental representations of E8,
it should not be a vertex on the same line as
the pentagonal bipyramid axis.
It should represent a vertex creating a triangle
whose base is one of the sides of the pentagon
and whose top is near one of the bipyramid-peak-vertices,
to which it is connected by a line.
To produce a symmetric figure,
the vertex must be reproduced in 5 copies, one over
each of the 5 sides of the pentagon.
Then, for the entire figure to be symmetric,
it must form an icosahedron.
The binary icosahedral group {2,3,5} is of order 120.

Another way to look at it is:

The graded sequence
248
248/\248
248/\248/\248
248/\248/\248/\248
248/\248/\248/\248/\248
has symmetry Cy(5) of order 5 for cyclic permutations,
but since 248/\248/\248/\248/\248 is fixed
by its relation to 3,875/\3,875/\3,875,
do not use Hodge duality on the 248 graded sequence.

The graded sequence
3,875
3,875/\3,875
3,875/\3,875/\3,875
has symmetry Cy(3) of order 3 for cyclic permutations,
but since 3,875/\3,875/\3,875 is fixed by
its relation to 248/\248/\248/\248/\248,
do not use Hodge duality on the 3,875 graded sequence.

The graded sequence
147,250
147,250/\147,250
has symmetry Cy(2) of order 2 for cyclic permutations,
but since 147,250/\147,250 is fixed by
its relation to 248/\248/\248/\248/\248,
do not use Hodge duality on the 147,250 graded sequence.

The +/- signs for the D5 half-spinors inherited
from E6 through E7
have symmetry of order 2.

Since the dimension of E8 is 248 = 120 +128,
the sum of the 120-dimensional adjoint representation of D8
plus
ONE of the 128-dimensional half-spinor representations of D8,
there is a choice to be made as to
which of the two half-spinor representations of D8 are used.
As they are mirror images of each other,
that choice has a symmetry of order 2.

Therefore:
the total symmetry group is of order 5x3x2x2x2 = 120,
the symmetry of the binary icosahedral group {2,3,5},
with 3 symmetries Cy(5), Cy(3), Cy(2).

It corresponds, by the McKay correspondence, to the E8 Lie Algebra.

What are the relations between 6,899,079,264
and 248/\248/\248/\248/\248, 3,875/\3,875/\3,875,
and 147,250/\147,250 ?

6,899,079,264 = 248/\248/\248/\248/\248 -
- 2 x 248/\248/\248/\248 -
- 2 x (248/\248/\248/\248 - 2 x 248/\248/\248 -
- (248/\248/\248 - 2 x 248/\248) - 248/\248) -
- 2 x 248/\248/\248
- (248/\248/\248 - 2 x 248/\248) - 248 =

= 248/\248/\248/\248/\248 -
- 4 x 248/\248/\248/\248 -
+ 3 x 248/\248/\248 -
- 248 =

= 7,506,861,544 -
- 4 x 153,829,130 +
+ 3 x 2,511,496 -
- 248 =

= 7,506,861,544 -
- 615,316,520 +
+ 7,534,488 -
- 248 = 6,899,079,264

6,899,079,264 = 3,875/\3,875/\3,875 - 18 x 248/\248/\248/\248 -
- 3 x 3,875/\3,875 +
+ 3 x 147,250 -
- 20 x 248 -
- 6 x 27 - 3 x 8 =
= 9,690,084,625 - 18 x 153,829,130 -
- 3 x 7,505,875 +
+ 441,750 -
- 4,960 -
- 162 - 24 =
= 9,690,084,625 - 2,768,924,340 -
- 22,517,625 +
+ 441,750 -
- 4,960 -
- 162 - 24 = 6,899,079,264

6,899,079,264 = 147,250/\147,250 - 25 x 248/\248/\248/\248 -
- 38 x 248/\248/\248 -
- 31 x 248/\248 -
- 55 x 248 - 128 -27 =
= 10,841,207,625 - 25 x 153,829,130 -
- 38 x 2,511,496 -
- 31 x 30,628 -
- 55 x 248 - 128 - 27 =
= 10,841,207,625 - 3,845,728,250 -
- 95,436,848 -
- 949,468 -
- 13,640 -
- 155 = 6,899,079,264

In the relations, the 8 dimensional representation of D4 was also used.

J. F. Adams has written a paper entitled The Fundamental Representations of E8
published in Contemporary Mathematics, Volume 37, 1985, 1-10,
and reprinted in The Selected Works of J. Frank Adams, Volume 2,
edited by J. P. May and C. B. Thomas, Cambridge 1992, pp. 254-263.

The 8 fundamental E8 representations are

```
                                              147250
                                                 |
248 - 30380 - 2450240 - 146325270 - 6899079264 - 6696000 - 3875
```

In his paper, Adams denotes the three at the ends as follows:

248 is denoted by alpha, which I will write here as a

3875 is denoted by beta, which I will write here as b

147250 is denoted by gamma, which I will write here as c.

Adams denotes the kth exterior power by lambda^k
which I will write here as /\k
and he denotes the kth symmetric power by sigma^k
which I will write here as Sk .
Also, here I write (x) for tensor product.

J. F. Adams says:

"... we can allow ourselves to construct new representations
from old by taking exterior powers, as well as tensor products
and Z-linear combinations ...

... seven of the eight generators for the polynomial ring R(E8)
may be taken as

a, /\2 a , /\3 a , /\4 a , b , /\2 b , c .

The eighth may be taken either as /\5 a , or as /\3 b , or as /\2 c .

The corresponding argument for Dn would say that one should begin
by understanding three representations of Dn = Spin(2n) ,

namely the usual representation on R^2n and the two half-spinor representations delta+ , delta- .
This we believe, so perhaps we can accept the analogue for E8 .

...

we must get back to business and construct b and c .

...

To give explicit formulae we must agree on a coordinate system.
The group E8 contains a subgroup of type A8 .
...
As a representation of A8 , the Lie lagebra L(E8) splits to give

L(E8) = L(A8) + /\3 + /\3* .

Let e1, e2, ... , e9 be the standard basis in the vector space V = C9 on which A8 acts
...
We now introduce the element

v_ik = SUM(j) (ei* ej* ek* (x) ej ek* + ej ek* (x) ei* ej* ek*)

in the symmetric square S2(a) in a (x) a , where a = L(E8) .
...
THEOREM 4.
(a) The representation S2(a) of E8 contains a unique copy of b .
(b) This copy of b contains the elements v_ik .
(c) For i =/= k the elements v_ik are eigenvectors corresponding to extreme weight of 8 .
(d) In particular (with our choice of details) v_gl is an eigenvector corresponding to the highest weight of b .
...
Theorem 4 allows us to realize b as the E8-submodule of S2(a) generated by v_91 (or any other v_ik with i =/= k .)
...
In fact, S2(a) contains a trivial summand ... and also an irreducible summand of highest weight ...
It turns out that the remaining summand has dimension 3,875,

which is precisely the dimension of b .

We now introduce the element

w_k = SUM(i) ei ek* (x) v_ik

= SUM(i,j) ei ek* (x) (ei* ej* ek* (x) ej ek* + ej ek* (x) ei* ej* ek*)

in a (x) b in   a (x) S2(a) in a (x) a (x) a .

...

THEOREM 5.
(a) The representation a (x) b of E8 contains a unique copy of c .
(b) This copy of c contains the elements w_k .
(c) The elements w_k are eigenvectors corresponding to the extreme weights of c .
(d) In particular (with our choice of details) w_1 is an eigenvector corresponding to the highest weight of c .
...
we may realize c as the E8-submodule of a (x) b generated by w_1 (or any other w_k).
...
In fact, a (x) b contains an irreducible summand of highest weight ... and the remaining summand has too small a dimension to contain two copies of c .   ...".

Adams then gives proofs of the theorems,
involving such things as looking at the Lie algebra L(E8) as
a representation of D8 as which it splits to give
L(E8) = L(D8) + delta-
where delta- is a half-spinor representation of D8.

One of the things that I think that I get out of Adams's paper is
that it seems to me that in order to get 3875 and 147250
you have to look not only at the 248 representation of E8 but
also at representations of some subgroups of E8
such as D8 etc.

Here are some excerpts from 4 slides of a talk by David Vogan (which has some simplifications/omissions of technicalities) and from part of a wwwliegroupsorg web page

- **What's a Lie group?**
  - A continuous family of symmetries.
- **How many Lie groups are there?**
  - One for every regular polyhedron.
- **Which one is E_8?**
  - The one for the icosahedron.
- **What's a group representation?**
  - A way to change under symmetry.
- **What's a character table?**
  - A description of all the representations.

E_8: $453,060$ vertices $\rightsquigarrow$ pieces of 240-dimensional flag variety.

How many Lie groups are there?

One for every regular polyhedron.

- 2D polygons: classical groups.
- Tetrahedron: E_6, dimension 78.
- Octahedron: E_7, dimension 133.
- Icosahedron: E_8, dimension 248.

Which one is E_8?

The one for the icosahedron.

There are three different groups called E_8, each one 248-dimensional and wonderfully complicated.

- Compact E_8. Characters computed by Weyl in 1925.

  In atlas shorthand, encoded by $\left(1\right)$.

  (Which hides deep and wonderful work by Weyl.)
- Quaternionic E_8. Characters computed in 2005.

  In atlas shorthand, a 73410×73410 matrix. One entry:

  $$3q^{13}+30q^{12}+190q^{11}+682q^{10}+1547q^{9}+2364q^{8}+2545q^{7}$$
  $$+2031q^{6}+1237q^{5}+585q^{4}+216q^{3}+60q^{2}+11q+1$$
- Split E_8. This is the tough one.

complex group E_8 has three real forms: the compact form, the quaternionic form (K=$A_1 \times E_7$) and the split form (K=D_8). Each of these gives rise to a different kind of real group of type E_8. Their blocks, and their sizes, are given by the `blocksizes` command of `atlas`:

	compact	quaternionic	split
compact	0	0	1
quaternionic	0	3,150	73,410
split	1	73,410	453,060

The last row means that the split group has 3 blocks (at infinitesimal character rho), of sizes 1, 73,410 and 453,060 respectively. (The columns give the group on which the Vogan-dual block lives.)

Here is some material from www.aimath.org/E8/

"... **The size of the answer**

The result of the E_8 calculation is a matrix, or grid, with 453,060 rows and columns.
There are 205,263,363,600 entries in the matrix, each of which is a polynomial.
The largest entry in the matrix is:

$$152 q^{22} + 3{,}472 q^{21} + 38{,}791 q^{20} + 293{,}021 q^{19} + 1{,}370{,}892 q^{18} + 4{,}067{,}059 q^{17} + 7{,}964{,}012 q^{16} + 11{,}159{,}003 q^{15} + 11{,}808{,}808 q^{14} + 9{,}859{,}915 q^{13} + 6{,}778{,}956 q^{12} + 3{,}964{,}369 q^{11} + 2{,}015{,}441 q^{10} + 906{,}567 q^{9} + 363{,}611 q^{8} + 129{,}820 q^{7} + 41{,}239 q^{6} + 11{,}426 q^{5} + 2{,}677 q^{4} + 492 q^{3} + 61 q^{2} + 3 q$$

If each entry was written in a one inch square,
then the entire matrix would measure more than 7 miles on each side.
... The computation was completed on January 8, 2007.
Ultimately the computation took 77 hours of computer time,
and **60 gigabytes to store the answer in a highly compressed form**.

... **By way of comparison, a human genome can be stored in less than one gigabyte**. For a more down to earth comparison,
60 gigabytes is enough to store 45 days of continuous music in MP3-format. ...

... Some other facts about the answer
Size of the matrix: 453,060
Number of distinct polynomials: 1,181,642,979
Number of coefficients in distinct polynomials: 13,721,641,221
Maximal coefficient: 11,808,808
Polynomial with the maximal coefficient: $152q^{22} + 3{,}472q^{21} + 38{,}791q^{20} + 293{,}021q^{19} + 1{,}370{,}892q^{18} + 4{,}067{,}059q^{17} + 7{,}964{,}012q^{16} + 11{,}159{,}003q^{15} + 11{,}808{,}808q^{14} + 9{,}859{,}915q^{13} + 6{,}778{,}956q^{12} + 3{,}964{,}369q^{11} + 2{,}015{,}441q^{10} + 906{,}567q^{9} + 363{,}611q^{8} + 129{,}820q^{7} + 41{,}239q^{6} + 11{,}426q^{5} + 2{,}677q^{4} + 492q^{3} + 61q^{2} + 3q$
Value of this polynomial at q=1: 60,779,787
Polynomial with the largest value at 1 which we've found so far: $1{,}583q^{22} + 18{,}668q^{21} + 127{,}878q^{20} + 604{,}872q^{19} + 2{,}040{,}844q^{18} + 4{,}880{,}797q^{17} + 8{,}470{,}080q^{16} + 11{,}143{,}777q^{15} + 11{,}467{,}297q^{14} + 9{,}503{,}114q^{13} + 6{,}554{,}446q^{12} + 3{,}862{,}269q^{11} + 1{,}979{,}443q^{10} + 896{,}537q^{9} + 361{,}489q^{8} + 129{,}510q^{7} + 41{,}211q^{6} + 11{,}425q^{5} + 2{,}677q^{4} + 492q^{3} + 61q^{2} + 3q$
Value of this polynomial at q=1: 62,098,473
...

...”.

P-adic Physics

P-adic Physics is useful in understanding the String Theory of R8 Universal Consciousness with interpretation of Strings as World-Lines of Many-Worlds.

At our scale, an elementary particles such as a lepton or quark is seen as a Vortex Cloud of virtual sea particle-antiparticle pairs whose objective form is that of a Kerr-Newman Black Hole whose identity is determined by a single Planck-scale Valence Particle whose past and future history path describes a World-Line.

As a 1-dim String World Line is perturbed (or vibrates) in the natural 26-dimensional space of Bosonic String Theory, it sweeps out a 2-dim String World Sheet, but the Line/Sheet structure breaks down at the Planck scale, at which scale P-adic structures emerge.

Lee Brekke and Peter G. O. Freund in Physics Reports 233 (1993) 1 - 66 say:
"... The boundary of the ordinary open string world sheet is the real line.
Along with the usual open string, one can consider p-adic open strings whose world sheet has as boundary the p-adic line instead (the points on this boundary are labelled by p-adic numbers rather than real numbers).
The world sheet itself is then no longer a continous manifold but becomes a discrete homogeneous Bethe lattice, or Bruhat-Tits tree of incidence number p + 1.
The p-adic strings thus correspond to specific discretizations of the world sheet.
Studying all these discretizations (one for each prime p) together, as suggested by number theory, sheds new light on the ordinary string ...[and]...a new type of string, the adelic string. ...".

B. Dragovich, A. Yu. Krennikov, S. V. Kozyrev, and I. V. Vololvich
in 0904.4205 [math-ph] say:
"... Suppose we have a physical ... system and we make measurements.
To describe results of the measurements, we can always use rationals.
To do mathematical analysis, one needs a completion of the field Q of the rational numbers. ... there are only two kinds of completions of the rationals ...
real R or p-adic Qp number fields, where p is any prime number with corresponding p-adic norm |x|p, which is non-Archimedean. ...
One can argue that at the very small (Planck) scale the geometry of the spacetime should be non-Archimedean. ...
Then rational, real or p-adic numbers should appear through a mechanism of number field symmetry breaking, similar to the Higgs mechanism. ...".

Goran S. Djordjevic, Branko Dragovich, and Ljubisa Nesic in hep-th/0105030 say:
"... the field of real numbers R and the fields of p-adic numbers Qp exhaust all number fields which may be obtained by completion of ... the field of rational numbers Q, and which contain Q as a dense subfield. ...
Real and p-adic numbers are unified in the form of adeles. An adele x is an infinite sequence x = (xoo, x2, · · · , xp, · · ·), where xoo is in R and xp are in Qp with the restriction that for all but a finite set S of primes p one has xp in Zp, where Zp = {a ∈ Qp : |a|p ≤ 1} is the ring of p-adic integers.

Componentwise addition and multiplication are natural operations on the ring of adeles A, which can be regarded as
A = UNION_S A(S) , A(S) = R x PRODUCT_pinS Qp x PRODUCT_pnotinS Zp
...
As adeles contain real and p-adic numbers, and consequently archimedean and nonarchimedean geometries, it has been natural to formulate adelic quantum mechanics, which provides a suitable framework for a systematic investigation of the above question. Uncertainty in space measurements
dx dy >= Lo^2 = hbar G / c^3 = 10^(-66) cm^2
which comes from ordinary quantum gravity considerations based on real numbers, restricts application of R and gives rise for employment of Qp at the Planck scale. It is also reasonable to expect that fundamental physical laws are invariant under interchange of R and Qp
...
one can introduce adelic path integral as an infinite product of ordinary and p-adic path integrals for all primes p. Intuitively, we regard adelic Feynman's functional integrals as path integrals on adelic spaces. ...
the formalism of ordinary and p-adic path integrals can be regarded as the same at different levels of evaluation, and the obtained results have the same form.
In fact, this property of number field invariance has to be natural for general mathematical methods in physics and fundamental physical laws.
In the present case it is mainly a consequence of the form invariance of the Gauss integral under interchange of R and Qp ...".

So, at the Planck scale,
p-adic tree structures describe the branching of the Worlds of the Many-Worlds and the non-loop terms of the Feynman Sum-Over-Histories Path Integral and **p-adic non-zero genus structures describe Path Integral loops.**

Chekhov, Mironov, and Zabrodin in Comm. Math. Phys. 125 (1989) 675-711
http://projecteuclid.org/DPubS/Repository/1.0/Disseminate?view=body&id=pdf_1&handle=euclid.cmp/1104179635
discuss P-adic geometry and Trees in the context of String Theory.
They say "... we propose a multiloop generalization ...
We consider the Bruhat-Tits tree which is an infinite homogeneous graph without cycles, each vertex being connected with exactly p + 1 neighbours by edges of unit length, as p-adic zero genus Riemann surface. ...
To produce p-adic Riemann surfaces of higher genera we should factorize the tree by some discrete (Schottky) groups. The surface obtained is the graph with cycles (their number is equal to the genus of the surface), the properties of this graph can be described by means of the ... reduced graph, which is the finite subgraph containing only the cycles with crosspieces between them. ...

2.2. Riemann Surface Over Qp and Bruhat-Tits Tree

... describe the construction of the Riemann surfaces [RS] over Qp field.
To obtain the extended zero genus QpRS one has to factorize PGL(2,Qp) by its maximal compact subgroup, that is PGL(2,Zp), being determined as 2 x 2 matrices with p-adic integer entries and invertible determinant in Zp. It is this homogeneous space that is called the Bruhat-Tits tree T. It is ... the connected infinite graph with no loops each vertex of T being connected with p + 1 neighbour vertices by edges. Obviously, any two vertices zl , z2 in the tree are connected by exactly one path z1 -> z2. We define the distance d(z1, z2) between these vertices to be the number of edges in the path z -> z2 . There are half-axes in the tree which are infinite subtrees with no branch points but with a single starting point (Fig. 2).

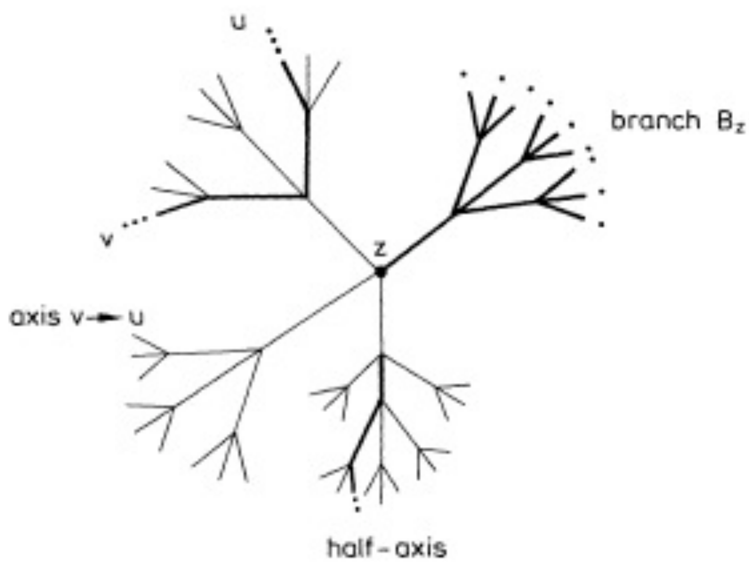


Fig. 2. Subgraphs in the tree: half-axis, axis $v \to u$ and branch B_z

equivalence classes ... for half-axes ...[are called]... rays.
... the tree T can be compactified by adding the set of "infinitely far points"
dT defined as the set of all rays.
In fact, dT can be canonically identified with P1(Qp).
On the other hand,
the PGL(2,Qp)-action can be naturally extended to dT from the tree T ,
so we shall consider dT as the boundary of a compactified tree T u dT
with PGL(2,Qp)-action on it.
In order to "coordinatize" P1(QP) we fix a point C (the "origin") in T.
This vertex corresponds to three half-axes starting at C
whose endpoints in dT = P1(Qp) are (0,1, oo), by definition.
Then we can identify P1(Qp) with Qp and after fixing C
only PGL(2,Zp)-freedom remains ...
C is the fixed point of PGL(2,Zp) and we describe the PGL(2,Qp)-action on T
manifestly as follows: PGL(2,Qp) acts on T = PGL(2,Qp) / PGL(2,Zp)
transitively and isometrically (i.e. the distances d( ) are conserved);
vertices correspond to g PGL(2,Zp) g^(-1) subgroups, g in PGL(2,Qp),
and edges correspond to g H g^(-1) subgroups, H = 2x2 matrix (a b / c d)
for a,b,c,d in Zp and ad not in M ,
where M is the unique maximal ideal in the ring Zp
(i.e. M = { x in Qp | |x|p < 1 }).
Let us define a branch Bz to be an entire subgraph of T with the only
boundary point z in the interior of T. The graph B is called entire if B \ dB is a
connected graph (Fig. 2) ...
we assume that the branches contain no cycles ... Then the set of rays contained in
Bz corresponds to an open domain dBz in dT and induce a natural topology on
Qp . Thus we have described the zero genus EQpRS .
...
Now introduce QpRS and EQpRS of higher genera.
We consider a Schottky group Y over Qp acting on T .
Then for any hyperbolic element y in Y the only y-invariant axis
in Y exists which is called the y-axis
(by definition, an axis is an infinite connected subtree with no branch vertices and
terminating vertices inside T (Fig. 2)).
Each element y acts by shifts along the corresponding y-axis, and for any
y conjugated to the element (q 0 / 0 1) 0 < |q|p = p(-ord_p(q)) < 1 , the shift is
equal to ord_p(q) . So the hyperbolic element has no invariant vertices and
edges. ...
the action of y on the whole tree is the shift along y axes defined by |q|p

with a simultaneous "rotation" around y-axes defined by the "phase" of q,
i.e. by q |q|p.
We define the Schottky tree T(Y), which is a union of axes of all elements of Y and crosspieces between them (crosspiece is the unique finite path which has common vertices but not edges with two chosen axes), or, equivalently,
a minimal connected subgraph containing the axes of all elements of Y
... Then T(Y) is in T, dT(Y) is in dT and T(Y)/Y = FR is a finite graph,
which is called the reduced graph ...
the reduced graph permits us to construct EQpRS from the Bruhat-Tits tree.
As the first step we consider the simplest example, namely, the torus.
We choose an element y generating Y1 in the form: y = (q 0 / 0 1) , 0 < |q|p < 1
The y-axis passes through the origin C, the fixed points u , v in Qp = dT are zero and infinity and any element of Y1 acts on the axis oo -> 0 by d-shifts,
d = 0 mod(ord_p(q)) (see Fig. 3a).

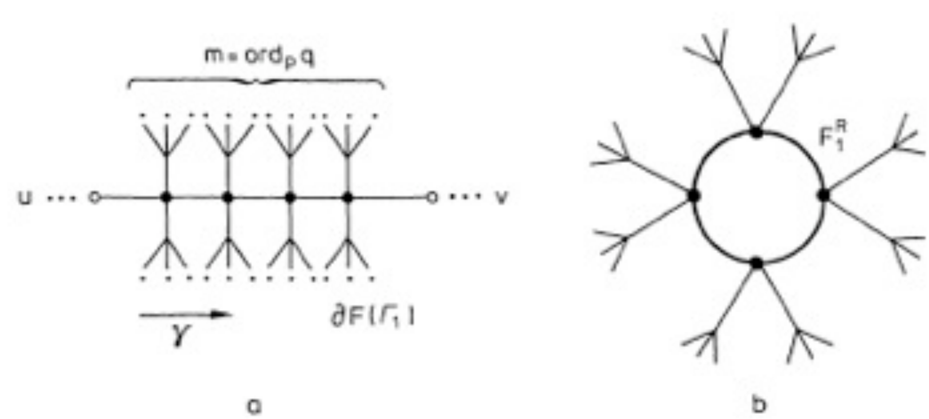


Fig 3 a–b. A typical fundamental domain of the Schottky group Γ_1 in the tree. **b.** The "extended" p-adic torus (in the case of $p=3$)

So the Schottky tree is a single axis,
FR1 is a cycle (ring) consisting of m = ord_p(q) edges.
To obtain EQpRS we factorize T by T1 : F1 = T / Y1 (Fig. 3b).
The result may be realized as a fundamental domain for Yγ glued into a ring.
Correspondingly, it induces the isomorphism dFF1 = d(T/Y1) = dT / Y1 .
So far we have seen that FR1 can be produced from F1 by truncating all branches with origins at the reduced graph. It appears to be the general procedure for the surface of arbitrary genus. The inverse operation is clear as well:
to construct EQpRS Fg = T/Yg it is necessary to draw the reduced graph
with a given number of loops (this number g is equal to genus of the surface)
and after that to add all the necessary branches with origins at the vertices

of the reduced graph in an evident way (Fig. 4a and b).

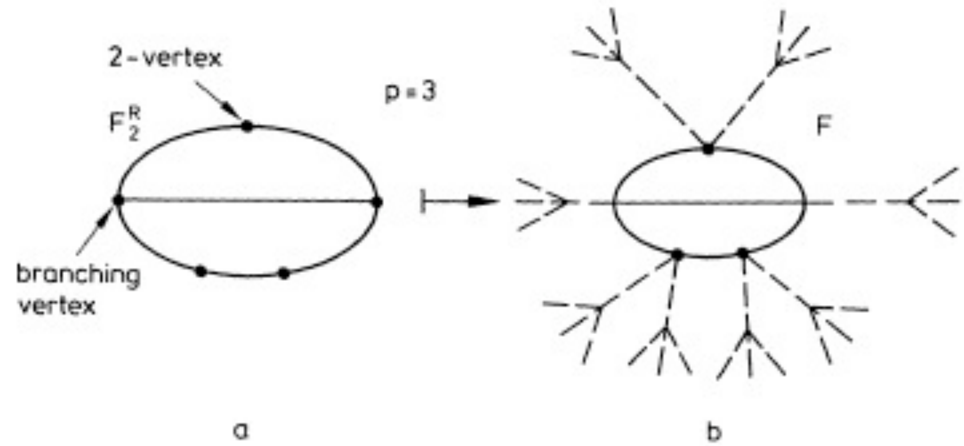


Fig. 4 a–b. A reduced graph for $g=2$. **b.** The whole factorized tree ($g=2$, $p=3$)

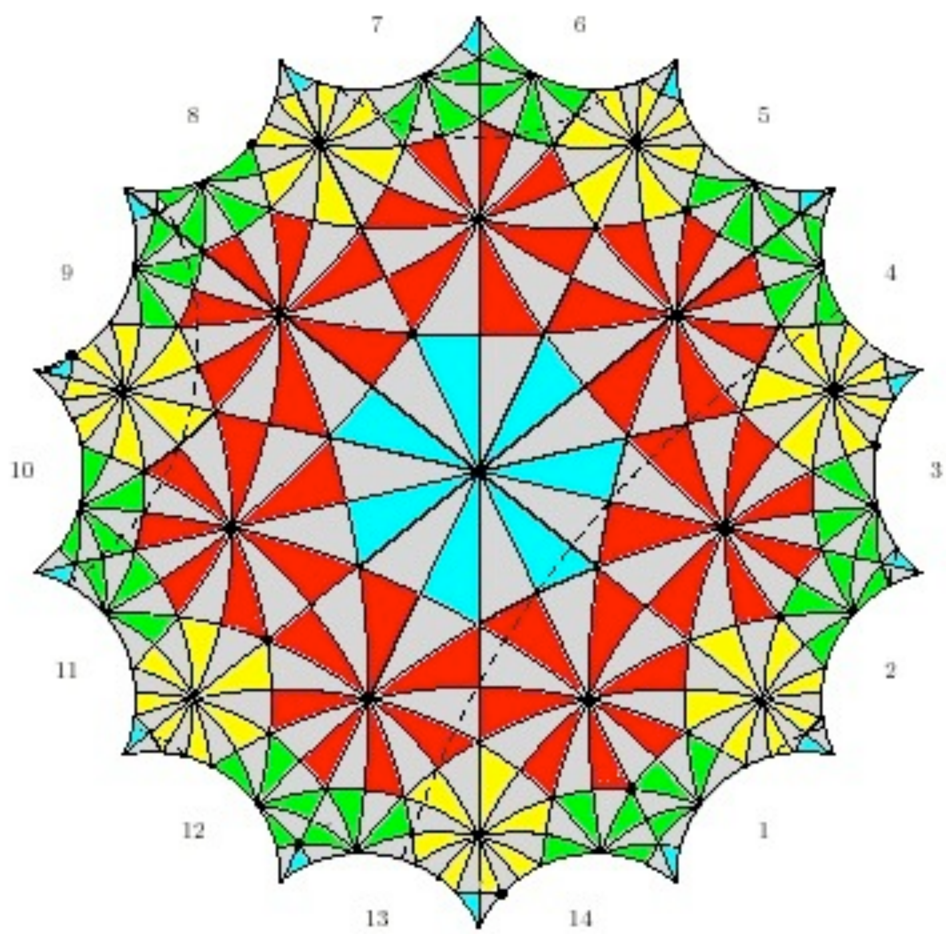
If one treats identity transformation as the trivial Schottky group Y0,
then FQ is merely a single vertex (denoted above as C), and T/Y0 = T.
Now one can see that the reduced graph consists of 3g - 3 or less segments
[segment in FRg is the line containing only 2-vertices
and connecting two branching vertices in FRg (see Fig. 4a)].
We denote si the lengths of these segments. In fact, they are "the moduli" of the corresponding p-adic surface. Strictly speaking, these parameters are p-adic orders of the moduli of an algebraic curve, but we shall call them merely moduli.
Thus FRg provides a good description of the moduli space of Riemann surfaces over Qp . ...".

It is interesting to have intuitive visualization of the p-adic tree structures that are fundamental for the physics (including the World-Line String Theory that gives the Bohm Quantum Potential of the Universal Consciousness) at the Planck scale.

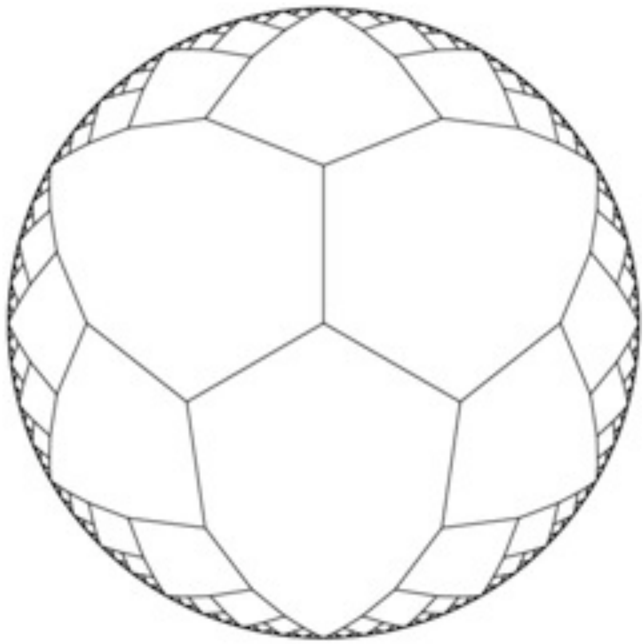
Anthony Phan has a web page at
http://www-math.univ-poitiers.fr/%7Ephan/exemples2.html
in which he says"... Paul Broussous and I had some discussions ...
he showed me how to have a visual perception of p-adic fields Qp
(where p is some prime number):
it may be represented thru some graphical representation of P1(Qp)
which is this field compactified by a point at infinity,
itself being the boundary of some infinite tree. ...".

Paul Broussous has a web page at
http://www-math.univ-poitiers.fr/~broussou/recherche.htm
in which he says in part (this is a google automated translation from French with some modification by me)
"... In the same way that one can visualize
the projective line over the field of real numbers as a circle,
the projective line on the field Qp of p-adic occurs naturally
as the set of "pieces" of a uniform tree of valence p + 1 . ...".

The idea is that the p-adic tree starts in the center of a circle
and branches out toward the edge of the circle,
with the edge of the circle being considered to be infinity
as is done in hyperbolic geometry of the Poincare disk,
an example being the Klein Configuration



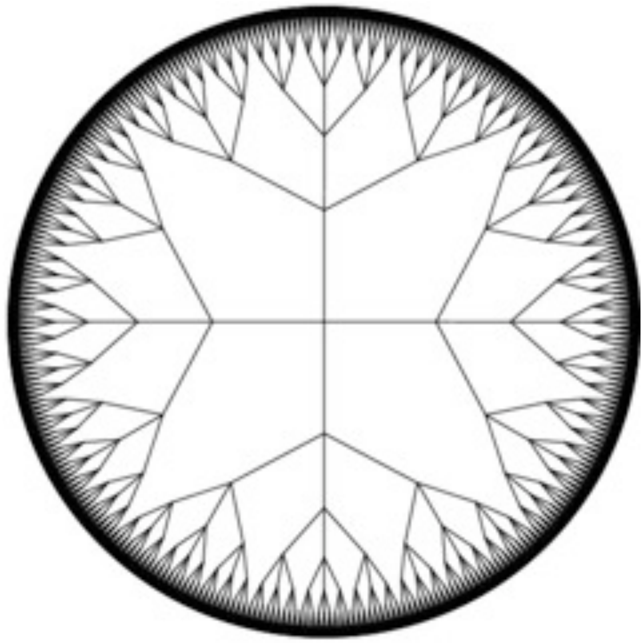
For the prime p = 2 the Broussous method gives



$P^1(\mathbb{Q}_2)$

which as you can see has p+1 = 2+1 = 3 trees from the center
with 2-fold branching to the edge of the circle (infinity in
the hyperbolic Poincare Plane type projection).
The binary 2-fold branching is like the YiJing
and
the binary + and - charges of electromagnetism
and
the binary system of fermion particles and antiparticles
and
the geometry of hypercubes.

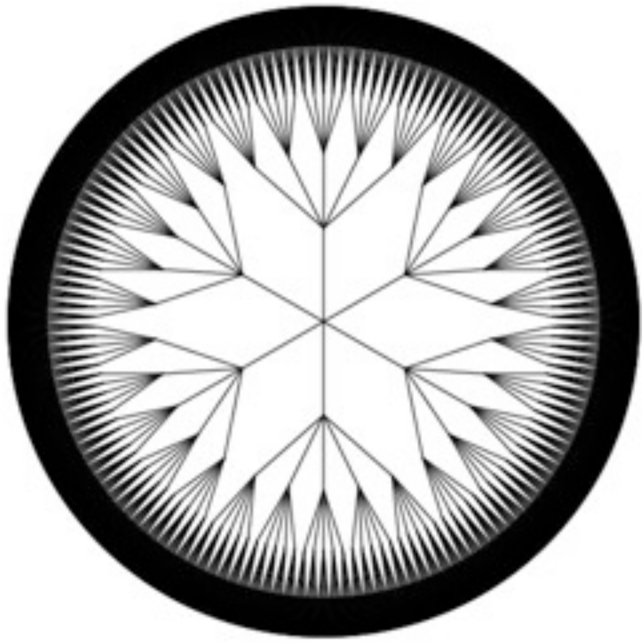
For the prime p = 3 the Broussous method gives



$P^1(\mathbb{Q}_3)$

which has p+1 = 3+1 = 4 trees from the center
with 3-fold branching to the edge of the circle (infinity in
the hyperbolic Poincare Plane type projection).
3-fold branching may correspond to such things as:
ternary sedenions (16-dim numbers multiplied 3 at a time);
the geometry of sub-hypercubes of a hypercube;
and
the Tai Hsuan Ching of 3x3x3x3 = 81 tetragrams of ternary lines
I have web material about it at
http://www.valdostamuseum.org/hamsmith/ichgene6.html#TaiHsuanChing

For the prime p = 5 the Broussous method gives



$P^1(\mathbb{Q}_5)$

which has p+1 = 5+1 = 6 trees from the center
with 5-fold branching to the edge of the circle (infinity in
the hyperbolic Poincare Plane type projection).
5-fold branching may correspond to such things as:
the icosahedron.

Note that these primes 2,3,5 were all important in the AMS Notices
paper "The p-adic Icosahedron"
http://www.staff.science.uu.nl/~corne102/publications/p-adic_icosahedron.pdf
which has similar pictures, and goes on to say:
"... Projecting the icosahedron from its center to a circumscribed sphere
maps each edge onto a part of a geodesic line on the sphere.
These geodesic lines intersect at vertices, midpoints of edges, or
barycenters of faces.
They provide a tessellation of the sphere by 120 triangles
with angles (pi/2 , pi/3 , pi/5) (Figure 3).

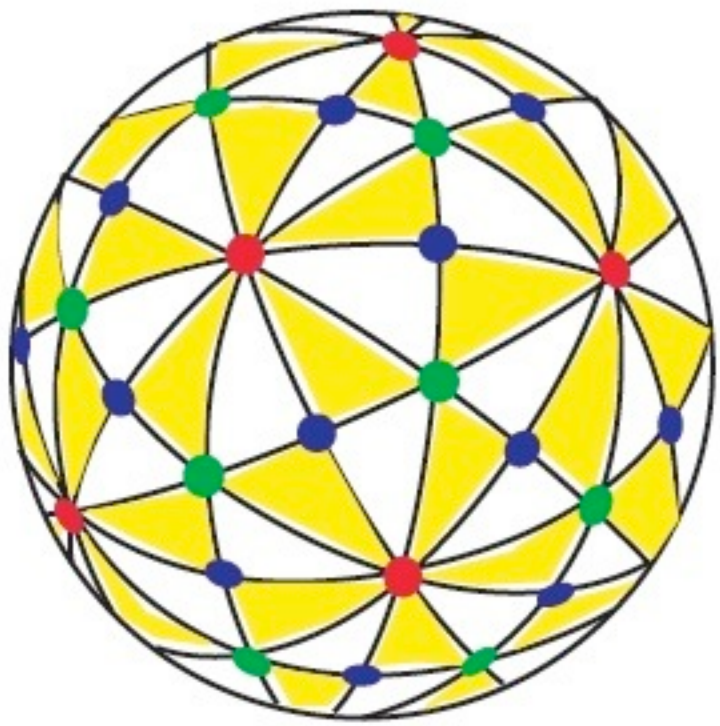


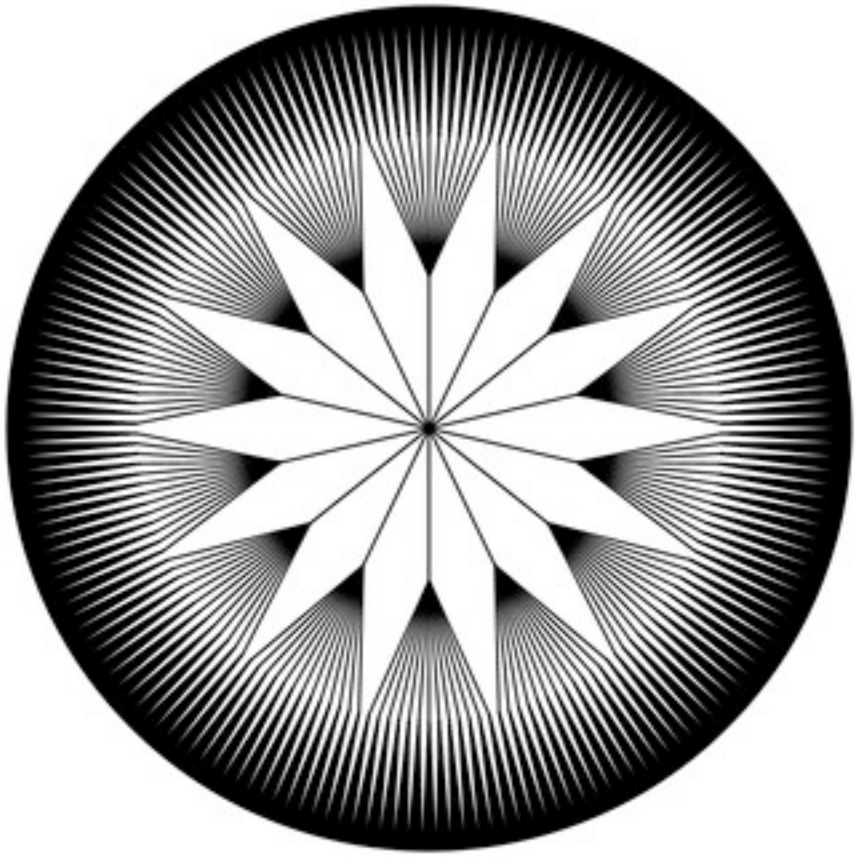
Figure 3. Icosahedral tessellation of the Riemann sphere.

... Let A5 denote the group of even permutations on five letters,
i.e., the simple group with 60 elements.
An icosahedral group is a copy of A5 embedded in PGL(2,C) ,
the automorphism group of the Riemann sphere
(= complex fractional linear transformations).
...

In order to find the p-adic analogue of the icosahedron ...
we have to introduce the p-adic analogue of the Riemann sphere,
which is an analytic structure on the projective line P1. The naive way
of “doing analysis with the p-adic metric on the coordinates” does not work
(because of total disconnectedness). ...
we will content ourselves with a description at the level of trees ...
The Bruhat-Tits tree T of PGL(2,Qp) ... edges emanating from any given vertex
are in one-to-one correspondence with Fp-rational points of P1.
The graph T is a regular (p + 1)-valent tree ...".

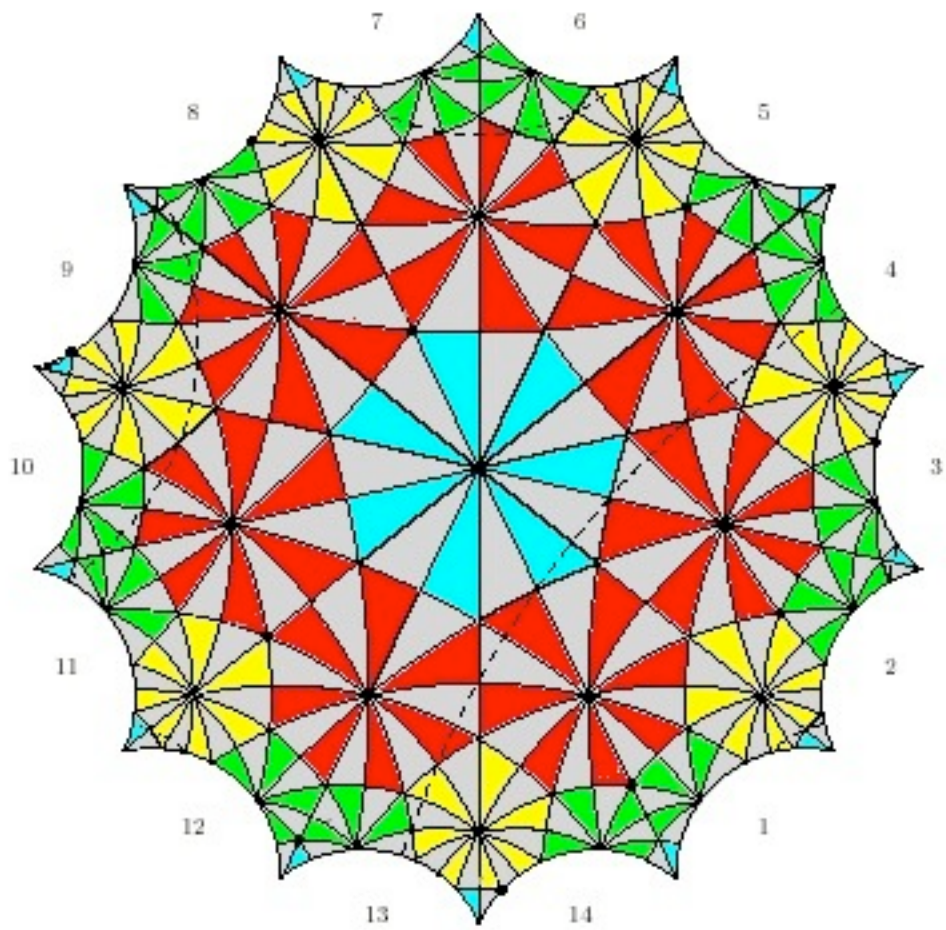
Notice that the icosahedral structure gives a correspondence between
the 120 triangles of the sphere and the 120 vertices of the 600-cell
and
that the icosahedral finite subgroup structure of the Riemann sphere
is similar to
the McKay correspondence between the icosahedral double group subgroup
structure of the 3-sphere and the E8 Lie algebra.

For the prime p = 13 the Broussous method gives



which has p+1 = 13+1 = 14 trees from the center
with 13-fold branching to the edge of the circle (infinity in the hyperbolic Poincare Plane type projection).

The 14 trees seem to correspond to the 14 sections of the Klein Configuration



which can be interpreted as describing the same physics model that I ususally describe in terms of E8 or Clifford Algebra. I have a paper about that at http://www.valdostamuseum.org/hamsmith/KleinQP.pdf
but
here I will just say that its Riemann surface is a 168-sheet covering of the sphere and so is related to the 168-element group PSL(2,7) which is a symmetry group of Octonion multiplications and Octonions are basic for E8.

13 is the only number between 5 and 89 that is both Prime and Fibonacci.

David C. Terr wrote a paper "Fibonacci Expansions and"F-Adic Integers"
http://www.fq.math.ca/Scanned/34-2/terr.pdf
in which he applies the ideas of p-adic (prime) theory
to his new idea of F-adic (Fibonacci) theory,
which should be of interest for 13 since it is both prime and Fibonacci,
as are 2, 3, and 5 of the icosahedral structures.
He says
"... The Fibonacci expansion of nonnegative integers ... may be thought of as
a base-T expansion, where T = (1/2)(1 + sqrt(5)) = 1.61803 is the golden mean).
...
the p-adic integers form a ring, but [some of] the F-adic integers do not.
My main result in this paper is that there is a 1-1 correspondence between
the F-adic integers and the points on a circle ... of circumference T^2 ...",

so

it is interesting that 13 (through its Fibonacci aspect) is related
to the Golden Ratio (similar to 2, 3, 5 and icosahedral structure).