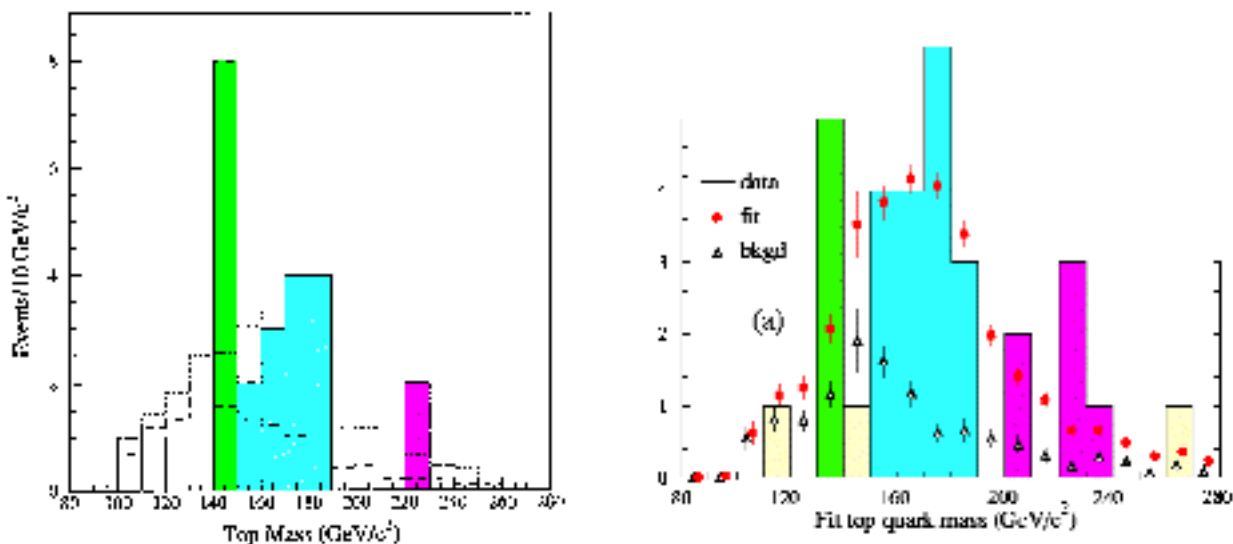
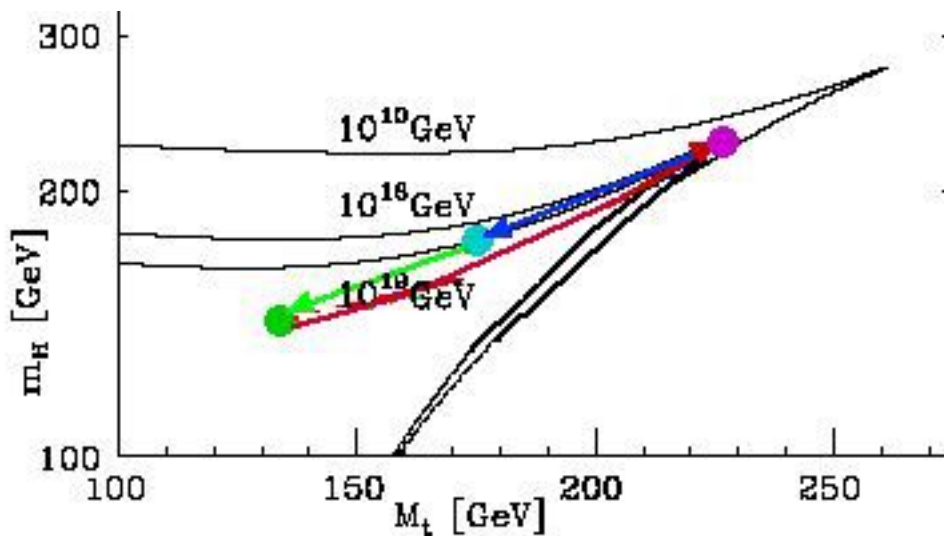


CDF and D0 observed 3 peaks in their T-quark data:



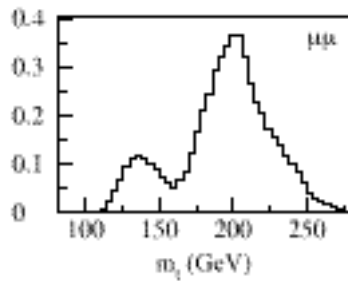
The middle (cyan) peak (around 175 GeV) is the one they initially identified as the T-quark.

The low peak (green) and the high peak (magenta) can be understood in terms of Froggatt's paper hep-ph/0307138:



in terms of a 3-part system of the T-quark, the Higgs, and the Triviality Bound.

The high and low peaks can be seen when you look closely at dilepton events, such as the D0 event Run 84395, Event 15530 (mu mu) described in [the 1997 UC Berkeley PhD thesis of Erich Ward Varnes](#) and in hep-ex/9808029



as analyzed using the matrix-element weighting algorithm that, according to [hep-ex/9808029](https://arxiv.org/abs/hep-ex/9808029), "... is an extension of the weight proposed in [R.H. Dalitz and G.R. Goldstein, Phys. Rev. D45, 1531 (1992)] ...".

Close study of all 3 peaks might give useful information about not only the T-quark and the Higgs, but also our Vacuum itself.

Koichi Yamawaki in his paper at [hep-ph/9603293](https://arxiv.org/abs/hep-ph/9603293) describes T-quark condensate Higgs models (NJL and BHL) that seem to be related to the **low and **high** T-quark peaks**

and

Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at [hep-ph/0311165](https://arxiv.org/abs/hep-ph/0311165) describe a T-quark condensate models in **8-dimensional spacetime (with 4 compact dimensions) that seem to be related to the **middle** T-quark peak.**

In his paper, [Yamawaki](#) says in part:

"... tightly bound composite Higgs models such as ... top quark condensate ...[which have]... nontrivial short distance dynamics of the gauged Nambu-Jona-Lasinio (NJL) models (gauge theories plus four- fermion interactions)

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

... in which the critical phenomenon in the gauged NJL model [yield] a simple reason why the top quark can have an extremely large mass compared with other quarks and leptons. ... the four-fermion theory

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

in the presence of gauge interactions (gauged NJL model) can become renormalizable and nontrivial in sharp contrast to the pure NJL model without gauge interactions. ...

... The Higgs boson was predicted as a $t\bar{t}$ bound state ...

... the top quark can have a large mass, or more properly, why other fermions can have very small masses ... if only the top quark coupling is above the critical coupling, while all others [are] below it ...

The top quark condensate ... indeed yields a standard gauge symmetry breaking pattern ... to feed the mass of W and Z bosons. ...

The largest physically sensible (new physics scale) would be the Planck scale 10^{19} GeV at which we have a minimum value prediction $m_t = 145$ GeV ... with the pure NJL case ...

The BHL [Bardeen-Hill-Lindner] value is then given by $m_t = 218 \pm 3$ GeV, at 10^{19} GeV ... The Higgs boson was predicted as a $t\bar{t}$ bound state ... Its mass was also calculated by BHL through the full RG equation ... the result being ... $M_H = m_t \times 1.1$ at 10^{19} GeV ...[which gives]... $M_H = 239 \pm 3$ GeV for $m_t = 218 \pm 3$ GeV ...[and $M_H = 143$ GeV for $m_t = 130$ GeV which is reasonably close to the 145 GeV value of m_t]..."

In other words, [Yamawaki](#) indicates that

the NJL model describes the low T-quark peak around 145 GeV, which is reasonably close to the 130 GeV value of the D4-D5-E6-E7-E8 VoDou physics model

the BHL model describes the high T-quark peak around 218 GeV and

the BHL model Higgs / T-quark mass ratio of 1.1 is consistent with the 145.8 GeV / 130 GeV = 1.12 ratio of the D4-D5-E6-E7-E8 VoDou physics model.

Also, [Hashimoto, Tanabashi, and Yamawaki](#) in their paper at [hep-ph/0311165](#) say:

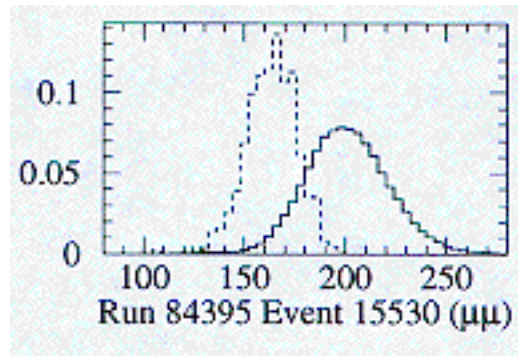
"... We perform the most attractive channel (MAC) analysis in the **top mode standard model with TeV-scale extra dimensions**, where the standard model gauge bosons and the third generation of quarks and leptons are put in $D(=6,8,10,...)$ dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We

then find that the top condensate can be the MAC for $D=8$... We predict masses of the top (m_t) and the Higgs (m_H) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ...

for ...[[Kaluza-Klein type](#)]... dimension... $D=8$... $m_t = 172-175$ GeV

and $m_H=176-188$ GeV ...".

The middle peak can be seen when you look closely at [the D0 dilepton event Run 84395, Event 15530 \(mu mu\)](#) described in [the 1997 UC Berkeley PhD thesis of Erich Ward Varnes](#) and in [hep-ex/9808029](#)



using the neutrino weighting algorithm. It has 3 jets. **If all 3 jets are included** (the solid line in the graph), energy around **200 GeV**, corresponding to the Standard Model Critical Point Truth Quark excited state at the **magenta dot**. **If only the 2 highest energy jets are included** (the dashed line in the graph), it has energy around **170 GeV**, corresponding to the 2-vacuum intermediate excited Truth Quark state at the **cyan dot**, and **the energy of the third jet would correspond to [decay down the blue curve along the Vacuum Stability bound of \$10^{19}\$ GeV](#)**.

Here are some more details from **Yamawaki's** paper at [hep-ph/9603293](#):

"... mass of all particles in the ... standard model (SM) ... is attributed to a single order parameter, the vacuum expectation value (VEV) of the Higgs doublet. Thus the problem of the origin of mass is simply reduced to understanding the dynamics of the Higgs sector...

... the situation very much resembles the Ginzburg-Landau (GL)'s macroscopic theory for

the superconductivity, the mysterious parts of which were eventually explained by the microscopic theory of Bardeen-Cooper-Schrieffer (BCS): The GL's phenomenological order parameter was replaced by the Cooper pair condensate due to the short range attractive forces.

... the sigma-model description by Gell-Mann and Levy (GML) works very well as far as the low energy (macroscopic) phenomena are concerned, while the deeper understanding of it was first given by Nambu and Jona-Lasinio (NJL)¹ based on the analogy with the BCS dynamics.

Nowadays people believe that essentially the same phenomena as described by the NJL paper takes place in the microscopic theory for hadrons, QCD.

In QCD the VEV of sigma ... 93 MeV, has been replaced by the quark-antiquark pair condensate ... an analogue of the Cooper pair condensate, formed by the attractive color forces.

The Nambu-Goldstone (NG) boson, the pion, is now a composite state of quark and antiquark.

This is actually the prototype of the dynamical symmetry breaking (DSB) due to composite order parameters like fermion pair condensates.

In fact Higgs sector in the SM is precisely the same as the sigma model except that ... the VEV of sigma ... 93 MeV ... is now replaced by the Higgs VEV ... 250 GeV ...

One is thus naturally led to speculate that there might exist a microscopic theory for the Higgs sector, with the Higgs VEV being replaced by the fermion-antifermion pair condensate ...

... tightly bound composite Higgs models were ... proposed based on the ... gauged NJL model (gauge theory plus four-fermion interaction)

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

within the framework of ladder Schwinger-Dyson (SD) equation. The gauged NJL model was shown to have a phase structure divided by a critical coupling (critical line) similarly to the NJL model, and have a large anomalous dimension due to strong attractive forces at relatively short distance or high energy. Such a system may actually be regarded as a theory with ultraviolet fixed point(s) in contrast to the asymptotic freedom. A remarkable feature of this dynamics is that the four-fermion interaction in four dimensions

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

may become renormalizable ... in a non-perturbative sense ... in sharp contrast to the pure NJL model ...

... We give a detailed comparison between the original formulation of Miransky-Tanabashi-Yamawaki (MTY) and another one of Bardeen-Hill-Lindner (BHL) ...

... [current masses](#) ... are entirely due to the Higgs VEV through the Yukawa coupling in the Glashow-Salam-Weinberg model and have nothing to do with the QCD dynamics ...

... QCD has no elementary order parameters. If the quark and gluon fields were order parameters, then the Lorentz invariance, color symmetry and charge symmetry would have been spontaneously broken in QCD in contrast to the reality. Then only possible order parameters are composite ones, variation of n-point Green functions or that of **local composite fields**. ... where ... a dynamical mass of quark, signals the spontaneous chiral symmetry breaking due to the QCD dynamics. We may define an "on-shell" dynamical mass m^* ... which is often called [constituent mass](#) (it also includes the effects of the explicit breaking due to the current mass). In contrast to the sigma model where ... there is no Yukawa coupling ... at Lagrangian level in QCD. However, we have an "induced" Yukawa vertex ... which is a "wave function" of [π] as a composite of [quark and antiquark] and is related to the dynamical mass ...

... As a low energy scale we take the scale parameter of QCD, Λ_{QCD} , which is typically of order 100 MeV - 1 GeV and actually characterizes the scale of the order parameters ... [pion force].. 93 MeV, ...[constituent light quarks]... 300 MeV or ...[$\Lambda_{\text{QCD}} = 250$ MeV]...

... **Nambu-Jona-Lasinio Model** ... the gauged NJL models (gauge theories plus four-fermion theories) ...

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

... encompass a variety of tightly bound composite Higgs models, such as .. top quark condensate ...[with anomalous dimension 2]... The NJL model is of course non-renormalizable and trivial theory, i.e., we cannot take the UV cutoff to infinity to have a sensible continuum theory, in contrast to the gauged NJL model ...

... **Gauged Nambu-Jona-Lasinio Model** ...

... **Tightly Bound Composite Higgs Models** ... There are a variety of tightly bound

composite Higgs models based on the gauged NJL model ... top quark condensate ... [with anomalous dimension 2]...

... **Top Quark Condensate** ... the ... extremely large ... top quark ... mass ... compared with mass of all other quarks and leptons and seems to suggest a special role of the top quark in the electroweak symmetry breaking, the origin of mass, and hence a strong connection with the Higgs boson itself. ... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction)

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

which triggers the top quark condensate. The Higgs boson emerges as a $t\bar{t}$ bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

are of $O(1)$, only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by **Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositeness condition**. BHL essentially incorporates $1/N_c$ sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at $1/N_c$ -leading order. Such effects turned out to reduce the above MTY value 250 GeV down to **220 GeV** ...

... **Top Quark Mass Prediction** ... the central part of the ... MTY ... model ... relat[es]... the dynamical mass of the condensed fermion (top quark) to the mass of W/Z bosons. ... the mass of W and Z bosons in the top quark condensate is generated via dynamical Higgs mechanism where ... 250 GeV ...determine[s] the IR scale of the model ... we could predict m_t by fixing ... [t]he decay constants of these composite NG bosons ...[about]... 250 GeV so as to have a correct m_W ... Actually, ... m_t ...[is] determine[d]... as a decreasing function of cutoff Λ . The largest physically sensible Λ (new physics scale) would be the Planck scale $\Lambda = 10^{19}$ GeV at which we have a minimum value prediction

$m_t = 145 \text{ GeV}$ Now in the gauged NJL model, QCD plus four-fermion interaction ...

[[In an 8-dimensional Kaluza-Klein version the four-fermion interaction is not needed.](#)]

...[f]or the Planck scale cutoff $\Lambda = 10^{19} \text{ GeV}$, we have $m_t = 250 \text{ GeV}$... This is compared with **the pure NJL case $m_t = 145 \text{ GeV}$** ...

... **RG Equation plus Compositeness Condition (BHL)** ... the BHL formulation of the top quark condensate, which is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ...

... **BHL versus MTY** ... MTY formulation is based on the nonperturbative picture ... On the other hand, BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale Λ ...[10^{19} GeV]... there must be a certain matching scale $\Lambda_{\text{Matching}}$ such that the perturbative picture (BHL) is valid for $\mu < \Lambda_{\text{Matching}}$, while only the nonperturbative picture (MTY) becomes consistent for $\mu > \Lambda_{\text{Matching}}$... However, thanks to the presence of a quasi-infrared fixed point, BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect $m_t = m_t(\text{BHL}) = 1/\tilde{A}^2 y_t(\mu = m_t) v = 1/(\sqrt{2}) y_{\text{bart}} v$ within 1-2%, where y_{bart} is the quasi-infrared fixed point given by $\text{Beta}(y_{\text{bart}}) = 0$ in ... the one-loop RG equation ... The composite Higgs loop changes y_{bart}^2 by roughly the factor $N_c/(N_c + 3/2) = 2/3$ compared with the MTY value, i.e., $250 \text{ GeV} \rightarrow 250 \times \sqrt{2/3} = 204 \text{ GeV}$, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. **The BHL value is then given by $m_t = 218 \pm 3 \text{ GeV}$, at $\Lambda = 10^{19} \text{ GeV}$.**

The Higgs boson was predicted as a $t\bar{t}$ bound state with a mass $M_H = 2m_t$ based on the pure NJL model calculation¹. Its mass was also calculated by **BHL through the full RG equation** ... the result being ... **$M_H / m_t = 1.1$**) at $\Lambda = 10^{19} \text{ GeV}$...".

Here are some more details from the paper of **Hashimoto, Tanabashi, and Yamawaki** at hep-ph/0311165:

"... The idea of the top quark condensate explains naturally the large top mass of the order of the electroweak symmetry breaking (EWSB) scale. In the explicit formulation of this idea often called the "top mode standard model" (TMSM), the scalar bound state of $t\bar{t}$ plays the role of the Higgs boson in the SM.

There are ... problems in the original version of the TMSM:

- We need to introduce ad hoc four-fermion interactions of the top quark in order to trigger the EWSB.
- ... if we take Λ to the Planck ... scale ...[s]uch a huge Λ ... causes a serious fine-tuning problem.

As a possible solution to these problems, following the line of an earlier attempt of the TMSM in the TeV-scale extra dimension [[Kaluza-Klein](#)] scenario Arkani- Hamed, Cheng, Dobrescu and Hall (ACDH) proposed an interesting version of such where the SM gauge bosons and the third generation of quarks and leptons live in the $D(= 6, 8, \dots)$ -dimensional bulk, while the first and second generations are confined in the 3-brane (4-dimensional Minkowski space-time).

[Compare [the D4-D5-E6-E7-E8 VoDou physics model](#). in which

- the first generation fermions move purely in [4-dimensional Minkowski Physical Space-Time \(PST\)](#),
- the second generation fermions have one foot in [4-dimensional Internal Symmetry Space \(ISS\)](#) and move from one of PST or ISS to the other, and
- the third generation fermions have two feet in PST but move in ISS

where 4-dim PST and 4-dim ISS are the products of [dimensional reduction](#) of [a high-energy 8-dimensional SpaceTime](#).]

Gauge interactions in higher dimensions than four become strong in a certain high-energy region. **Bulk gauge interactions are expected to trigger the top condensation without adding ad hoc four-fermion interactions, in contrast to the original version of the TMSM.**

However, the dynamics of bulk gauge theories was not concretely analyzed ...[by ACDH]... In particular, as it turned out **the bulk QCD coupling, which is the most relevant interaction for the top condensation, has an ultraviolet fixed point (UV-FP) or upper bound within the same \overline{MS} scheme of the truncated Kaluza-Klein (KK) effective theory as that ...[the work of ACDH]... was based on.** Thus, it is quite nontrivial whether the top condensation is actually realized or not.

... we have studied the dynamical chiral symmetry breaking (DxSB) in bulk gauge theories, based on the ladder Schwinger-Dyson (SD) equation. Switch ing off the electroweak interaction in the bulk, we then found that **the bulk QCD coupling can ... become sufficiently large to trigger the top condensation for ... $D = 8$.**

For the purpose of model building, we further need to study the effect of the bulk electroweak interactions: Since the bulk $U(1)Y$ interaction grows very quickly due to the power-like running behavior and reaches immediately its Landau pole Λ_{LY} , it may affect the most favored channel for condensate, i.e., the most attractive channel (MAC). We also need to study whether or not the prediction of the top mass agrees with the experiments.

... we demonstrate a possibility that the top condensate is actually the MAC even including all of the bulk SM gauge interactions. This is quite nontrivial, because inclusion of the strong bulk $U(1)Y$ interaction may favor the tau condensation rather than the top condensation. In order for only the top quark to acquire the dynamical mass of the order of the EWSB scale, the binding strength should exceed the critical binding strength ... only for the top quark ("topped MAC" or "tMAC"). Namely, **our scenario works only when... the binding strength... of the top ... condensate...[exceeds]... the critical binding strength ...[which in turn exceeds]... the binding strength ... of the ... bottom... and tau condensates at the scale Λ , ...** We refer to the scale Λ satisfying ...[those relationships]... as the tMAC scale Λ_{tM} .

For the MAC analysis, **we study binding strengths ... by using the one-loop renormalization group equations (RGEs) of dimensionless bulk gauge couplings.** It is in contrast to the analysis of ACDH where all of bulk gauge couplings are assumed equal (and strong enough for triggering the EWSB). In order to check reliability of our MAC analysis, we also study the regularization-scheme dependence of the binding strengths. We calculate gauge couplings in two prescriptions, the \overline{MS} scheme of the truncated KK effective theory and the proper-time (PT) scheme.

There are some varieties in the estimation of ... the critical binding strength ... The naive dimensional analysis (NDA) implies ... the critical binding strength ... [is about]... 1, while the ladder SD equation yields much smaller value ... the critical binding strength ... [about]... 0.1 . As the estimate of ... the critical binding strength ... increases ... the region of the tMAC scale gets squeezed. Even if we adopt the lowest possible value of ... the critical binding strength ... given by the ladder SD equation, we find that the tMAC scale does not exist for the simplest scenario with $D = 6$. On the other hand, **the tMAC scale does exist in $D = 8$ for the value of the ladder SD equation, $\Lambda_{tM} R = 3.5\text{--}3.6$, where the compactification scale R^{-1} is taken to be 1-100 TeV.** For $D = 10$, the MAC analysis significantly depends on the regularization scheme.

Once we obtain the tMAC scale tM , we can easily predict the top mass m_t and the Higgs mass m_H by using the renormalization group equations (RGEs) for the top Yukawa and Higgs quartic couplings, and the compositeness conditions at the scale $\Lambda = \Lambda_{tM}$. This is in contrast to the earlier approach ..[of ACDH]... where the composite scale Λ is treated as an adjustable free parameter and fixed so as to reproduce the experimental value of m_t .

Without such an adjustable parameter, we predict the top quark mass

$$m_t = 172 - 175 \text{ GeV for } D = 8 \text{ and } R^{(-1)} = 1-100 \text{ TeV.}$$

... We find that the value of m_t near the compactification scale $R^{(-1)}$ is governed by the quasi infrared fixed point (IR-FP) for the top Yukawa coupling y ... We also predict the Higgs boson mass as $m_H = 176 - 188 \text{ GeV}$... [which is close to the range of $1.1 m_t = 189 - 193 \text{ GeV}$]...

Thanks to the IR-FP property, the prediction for m_t and m_H is stable. ...

Let us consider a simple version of the TMSM with extra dimensions where the SM gauge group and the third generation of quarks and leptons are put in D -dimensional bulk, while the first and second generations live on the 3-brane (4-dimensional Minkowski space-time).

[Compare [the D4-D5-E6-E7-E8 VoDou physics model](#). in which

- the first generation fermions move purely in [4-dimensional Minkowski Physical Space-Time \(PST\)](#),
- the second generation fermions have one foot in [4-dimensional Internal Symmetry Space \(ISS\)](#) and move from one of PST or ISS to the other, and
- the third generation fermions have two feet in PST but move in ISS

where 4-dim PST and 4-dim ISS are the products of [dimensional reduction](#) of [a high-energy 8-dimensional SpaceTime](#).]

The D -dimensions consist of the usual 4- dimensional Minkowski space-time and extra ... spatial dimensions compactified at a TeV-scale $R^{(-1)}$. The number of dimensions D is taken to be even, $D = 6, 8, 10, \dots$, so as to introduce chiral fermions in the bulk. In order to obtain a 4-dimensional chiral theory and to forbid massless gauge scalars, we compactify extra dimensions on the orbifold ... [$T^{(D-4)} / Z^{((D-4)/2)}_2$]...

We emphasize that there is no elementary field for Higgs in our model. The chiral condensation of bulk fermions may generate dynamically a composite Higgs field, instead.

Hence we investigate RGEs of bulk gauge couplings including loop effects of the composite Higgs.

We expand bulk fields into KK modes and construct a 4-dimensional effective theory. ...

... We calculate the RGEs by using the UV-BCs ... and determine m_t and m_H through the conditions,

- $m_t = (v / \sqrt{2}) y(m_t)$,
- $m_H = v \sqrt{\lambda(m_H)}$...
- with $v = 246$ GeV.

We show results of m_t and m_H in Fig ... 8 ...

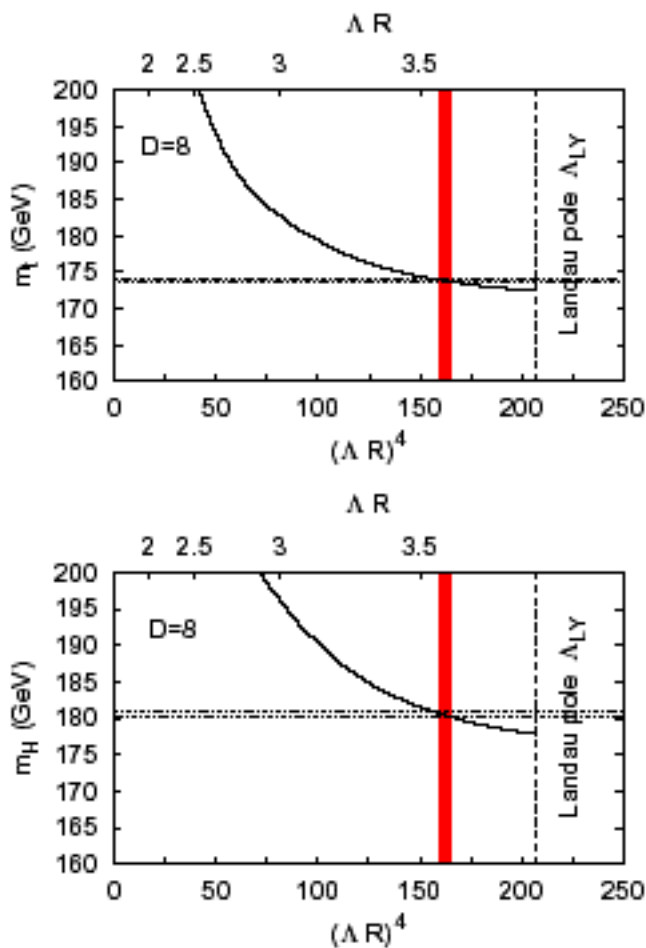


FIG. 8: Solutions m_t and m_H of Eq. (60) with the compositeness conditions (52) for $D = 8$, $R^{-1} = 10$ TeV. The dashed vertical line represents the Landau pole Λ_{LY} . The shaded region is the tMAC scale Λ_{tM} satisfying Eq. (41).

for $D = \dots 8 \dots$, $R^{-1} (= 10$ TeV for various values of the compositeness scale Λ the tMAC scale does exist only for $D = 8$ without much ambiguity, $\Lambda_{tM} R = 3.5-3.6$. Identifying Λ with tM, we depict the region of the tMAC scale for $D = 8$ by the shaded area in Fig. 8. ... For $D = 8$ we predict $m_t = 172 - 175$ GeV, and $m_H = 176 - 188$ GeV, for the range of the compactification scale $R^{-1} = 1-100$ TeV. The uncertainties ... also include error of $\alpha_3(M_Z) = 0.1172 \pm 0.0020$.

... the prediction ... for m_t is consistent with the reality, the MSbar mass $m_t = 164.7 \pm 4.9$

GeV which is calculated from the observed value of the pole mass, $174.3 \pm 5.1 \text{ GeV}$ our compositeness scale is fixed by the tMAC scale tM by requiring that the top quark condensation actually takes place, while other condensations do not. Hence the top mass as well as the Higgs mass is the prediction in our approach. ...

... the value ... is significantly smaller than that of the original TMSM in four dimensions which predicted $m_t \geq 200 \text{ GeV}$. Let us consider a simplified RGE for y neglecting the electroweak gauge interactions ... we find the quasi IR fixed point $y_{qIR}(\mu)$... decreases as ... $(D-4)$ increases at $\mu = R^{-1}$... As a result, the prediction of m_t with ... $[D-4]$... > 0 is substantially lower than that of the original TMSM with ... $D-4]$... = 0. ...

The mechanism is still operative even including the electroweak gauge interactions: In Fig. 10,

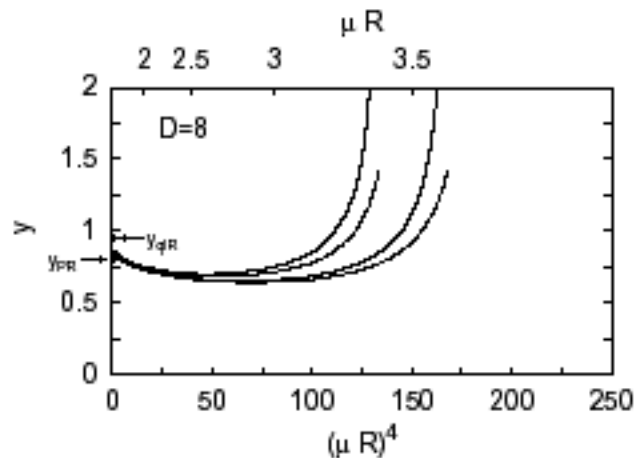


FIG. 10: RGE flows for the top Yukawa coupling y . We also show the quasi IR fixed point y_{qIR} and the PR fixed point y_{PR} at $\mu = R^{-1}$. The graph represents running of y for $D = 8$. $R^{-1} = 10 \text{ TeV}$ was assumed. We used the full one-loop RGE (51). The UV-BCs are $y(\Lambda) \rightarrow \infty$ (solid lines) and $y(\Lambda) = \sqrt{2}$ (dashed and dash-dotted lines) for two typical values of Λ .

we show the quasi IR fixed point and the behavior of y based on the full one-loop RGE with various boundary conditions at Λ . We also show the Pendleton-Ross (PR) fixed point ... As far as ... $D = \dots 8$... the value of the PR fixed point is smaller than that of the quasi IR fixed point ... The top Yukawa coupling at R^{-1} for $D = \dots 8$... is actually between ... the PR fixed point ... and ... the quasi IR fixed point ... for a sufficiently large top Yukawa, $y(\Lambda) \geq 1$, at high energy scale $(\Lambda R)^{D-4} \gg 1$... We note here that the actual prediction of m_t with $D = \dots 8$ is even smaller than the value expected from y_{qIR} .

We also comment that the predicted values of m_t and m_H would be stable thanks to these fixed points, even if the estimate of the tMAC scale were somewhat changed from ours for some reason. ... the lower value prediction of m_t than that of the original TMSM can also be understood as follows: Since KK modes of the top quark ($t^{(n)}$) as well as its zero

mode ($t^{(0)}$) contribute to the VEV v ... the condensate $\langle \bar{t}^{(0)} t^{(0)} \rangle$ is suppressed compared with the original TMSM and so is the top mass.

Now we discuss implication of our Higgs mass prediction ... The upper limit of m_H from radiative corrections in the SM is $m_H < 199$ GeV at 95% CL. The prediction ... is still below this upper limit. ...

... SUMMARY AND DISCUSSIONS

We have argued a viable top mode standard model (TMSM) with TeV-scale extra dimensions where bulk $SU(3) \times SU(2) \times U(1)$ SM gauge interactions (without ad hoc four-fermion interactions) trigger condensate of only the top quark, but not of other quarks and leptons.

In order for such a situation to be realized, the binding strength ... should exceed the critical binding strength ... only for the top quark (tMAC) ... The binding strengths ... were calculated by using RGEs for bulk SM gauge couplings. ... We then found that the region of the tMAC scale is squeezed out for $D = 6$... while it does exist for $D = 8$, $\Lambda = (3.5-3.6) R^{-(1)}$. We were not able to draw a reliable conclusion for $D = 10$ since the MAC analysis for $D = 10$ strongly depends on the regularization scheme.

For $D = 8$, we predicted the top mass m_t and the Higgs mass m_H : $m_t = 172 - 175$ GeV and $m_H = 176 - 188$ GeV, by using RGEs for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the tMAC scale tM . Our predictions are governed by the quasi IR-FP and hence are stable against varying the composite scale. The predicted values would not be changed so much, even if the region of the tMAC scale got wider than our estimate for some reason.

Why is the value ... [of m_t] ... significantly smaller than that of the original TMSM in four dimensions which predicted $m_t \geq 200$ GeV? The value of the top Yukawa coupling at the quasi IR-FP is ... suppress[ed] ... [by a] ... factor $2^{-(D-4)/2}$... [so].. the mass of the top quark decreases as the number of dimensions increases. ...

Many issues remain to be explored:

- 1) Our results on the tMAC scale are sensitive to the value of ... the critical binding strength ... Although we used the reference value of ... the critical binding strength ... in the approach of the ladder SD equation neglecting the effect of the compactification, it would be more preferable if we can determine ... the critical binding strength ... more reliably. For such a purpose, we should take into account effects of the compactification scale $R^{-(1)}$ which turned out not so small compared with the tMAC scale Λ_{tM} in our analysis. We also need running of bulk

gauge couplings beyond one-loop perturbation.

- 2) We incorporated only one composite Higgs doublet into RGEs, assuming other possible bound states such as vector/axial-vector bosons are irrelevant. In order to justify the assumption, we need to solve bound state problems in the bulk gauge theories. Once such a composite scalar exists, it should be a tightly bound state formed by strong short distance dynamics with large anomalous dimension. Such a system is expected to resemble the gauged Nambu-Jona-Lasinio (GNJL) model where the compositeness condition is explicitly formulated. Actually, as it happened in the 4-dimensional case, the pure gauge dynamics strong at short distance in our case can also induce strong four-fermion interactions which may become relevant operators due to large anomalous dimensions, $\gamma_m = D/2 - 1$
- 3) There are potential constraints on our model from precision electroweak measurements. The summation of KK modes below the cutoff Λ contributes to $\Delta\rho$ as $\Delta\rho = (\Lambda/R)^{D-6} (M_W/R)^2$. In our case with $\Lambda = \Lambda_{tM}$, $(\Lambda_{tM}/R)^{D-6} = 10$ for $D = 8$, we thus need to take $R^{-1} > O(10 \text{ TeV})$, which may be subtle about the fine tuning. ...
- 4) Masses of other quarks and leptons have not been dealt with in this paper. In the original TMSM, these masses are descended from the top condensate through ad hoc flavor-breaking four-fermion interactions. The origin of such four-fermion interactions will be highly hoped for in the present scenario.
- 5) Our scenario crucially relies on the short distance strong dynamics around the composite scale. We thus need a better-controlled theory in the UV-region. It would be interesting to study a deconstructed/latticized version of our model. ...".

Extra gauge field structure uncovered in the Kaluza-Klein framework,

Class. Quantum Grav. 3 (1986) L99-L105, by **N. A. Batakis** says:

"... In a standard Kaluza-Klein framework,

M4 x CP2 allows the classical unified description of an SU(3) gauge field with gravity.

However,

the possibility of an additional SU(2) x U(1) gauge field structure is uncovered.

... As a result, M4 x CP2 could conceivably accommodate the classical limit of a fully unified theory for the fundamental interactions and matter fields. ... [There are]... two generic possibilities ... for the enlargement of Einstein's framework, namely

- ... increase the number of spacelike dimensions ...[which]... is mainly exploited in the ...[ordinary]... Kaluza-Klein programme ...[in which]... the extra dimensions form a vertical 'internal' compact space of very small ... volume ... and ...
- ... allow the presence of torsion without upsetting the metricity of connections ... [which involves not only]... a torsion (totally within M4) ... in the context of the Einstein-Cartan theory ...[but also]... the ... mixed components of a torsion in the total space, namely components which are neither totally vertical nor completely horizontal. ... such a torsion creates a new and non-trivial possibility for the accommodation of unified theories in the KK framework ...[in a]... way in which an eight-dimensional manifold, locally of the form M4 x CP2, could ... accommodate the classical limit of a fully unified theory for the fundamental interactions and matter fields ...

...[In]... M4 x CP2 ... the groups **G1** and **G2** are **SU(3)** and **SU(2) x U(1)**, respectively.

The ... **G1 [SU(3)]** ... results from the well known identification of CP2 with the coset space **SU(3)/U(2)**.

... the **G2 ... SU(2) x U(1)** ...[has a]... Killing form is not zero but ... is degenerate, namely (- 1, - 1, - 1, 0). However, in view of the U(1) factor ... a non-degenerate metric (- 1, - 1, - 1, - 1) can be (and often is) defined on SU(2) x U(1). This possibility makes SU(2) x U(1) a perfectly acceptable G2 ...The metric is given by ...

$$g_{MN} = \left(\begin{array}{c|c} g_{\mu\nu}^0 + \kappa_1^2 \xi_P^a \xi_Q^b A^{(1)P}{}_\mu A^{(1)Q}{}_\nu g_{ab}^0 & \kappa_1 \xi_Q^a A^{(1)Q}{}_\mu g_{an}^0 \\ \hline \kappa_1 \xi_Q^a A^{(1)Q}{}_\nu g_{am}^0 & g_{mn}^0 \end{array} \right)$$

... and the connection 1-form $w^M{}_N$ is defined as $w^M{}_N = w^0M{}_N + K^M{}_N$ with $w^0M{}_N$ the Riemannian connection of the metric [shown immediately above]... and the contorsion $K^M{}_N$ defined ... in terms of the torsion ... $T^m{}_{ab} = k_2 \theta^m{}_I F^{(2)} I^*{}_{ab}$ where * denotes the M4 dual, k_2 is a constant and $\theta^m{}_I$ is a vielbein (employed to change the group index I to the [CP2]... index m) such that $g_{mn} = \theta^m{}_I \theta^n{}_J = -g_{IJ}$... The Riemann scalar curvature is then given by an equation similar to ...

$$R^{(5)} = \overset{1}{R}^{(5)} - 2K^{MN}{}_{M;N} - K^{MN}{}_N K_{ME}{}^E + K_{MNE} K^{ENM} \dots$$

$$R^{(5)} = \overset{1}{R}^{(5)} - \frac{1}{4} \kappa_2^2 (F^{(2)})^2 + \text{surface terms} \dots$$

$$R^{(5)} = R^{(4)} - \frac{1}{4} \kappa_1^2 (F^{(1)})^2 - \frac{1}{4} \kappa_2^2 (F^{(2)})^2 + \text{surface terms.}$$

... and the Bianchi identities hold ... **The resulting Einstein-Hilbert action ... when expressed totally within M4 will, besides gravity (with a cosmological constant), contain the two gauge fields [SU(3) and SU(2) x U(1)], with the relative scales between the three parts set by ... constants k_1 and k_2 as in ...**

the Einstein-Hilbert action reduces in four dimensions (with κ_0 a constant) to

$$I = \kappa_0^2 \int (-g^{(4)})^{1/2} d^4x \left(-R^{(4)} + \frac{1}{4} \kappa_1^2 (F^{(1)})^2 + \frac{1}{4} \kappa_2^2 (F^{(2)})^2 \right). \quad (17)$$

... We recall

- that **A(1) [of SU(3)] has been introduced at the level of the metric via the Kaluza ansatz ...[and] has a role for the coset space ...[CP2]... analogous to that of the Lorentz (or Poincare) group for ordinary spacetime,**
- while **A(2) has been introduced directly through a field strength $F(2)$ at the level of the connection. In view of the Bianchi identities for the manifold [M4 x CP2] ... $F(2)$ will have a well defined and conserved energy-momentum tensor. However, its gauge group structure is apparently not mandatory. What our construction has shown is that the geometry allows a maximal gauge group structure ... The corresponding [$G_2 = SU(2) \times U(1)$] gauge symmetry is**

apparently unprotected, in contrast to the G1 gauge symmetry.

These results are obviously desirable in view of the ... association we seek for the two gauge fields with the strong and electroweak interactions. We also observe that we have exhausted the generic possibilities for the introduction of interaction fields into the geometry: besides the metric and a general metric connection, there is no other independent intrinsic geometric structure available within our framework. Thus, the following geometric picture seems to be emerging.

- The gravitational and $SU(3)$ gauge field potentials must be considered as more fundamental and they completely specify the metric - essentially they are the metric of $M4 \times CP2$. If no torsion exists, a symmetric metric connection is uniquely defined from this metric and $M4 \times CP2$ would then exhibit a complete left-right symmetry.
- However, ... [$SU(2) \times U(1)$] symmetry will break ... with the introduction of torsion. The mixed [torsion] components of the ... [$SU(2) \times U(1)$ symmetry.].. will be associated with the spin-1 field $F(2)$ with a (possibly broken) $SU(2) \times U(1)$ gauge symmetry.
- ... [The torsion]... components totally within $M4$ or $CP2$ could accommodate matter fields in the form of, respectively, spin density and energy-momentum density condensates, with mechanisms analogous to those already known ...".

In the paper, Batakis does not discuss "... the introduction of matter fields ... or related ... issues ...".

With respect to spinor fermions and spin structure, even though $CP2$ is not a spin manifold, it is a spin_c (complex) manifold as described in the book *Spin Geometry* by Lawson and Michelsohn (Princeton 1989) particularly page 392.

Also, the book *Analysis, Manifolds, and Physics* by Choquet-Bruhat and DeWitt-Morette with Dillard-Bleick (North-Holland 1982 rev ed) says at page 418:

"... In cases where no spinor structure exists, one may define a generalized spin structure. In this scheme one makes topological room for a "spin structure" by mixing in with the group of a spin bundle an "internal" symmetry group, which is inextricably involved in the generalized spinor transformation rule.

It seems to me that the KK model of Batakis may well have a realistic generalized spin structure, and that if the spinor fermion spectrum is inherited from the 8-dim Dirac structure of $Cl(1,7)$ prior to breaking the 8-dim spacetime into $M \times CP2$ at lower energies, then the Batakis structure may turn out, upon further development, to be very similar to [the D4-D5-E6-E7-E8 VoDou Physics model](#) (which is in agreement with experiments).

An interesting thing about the 1986 paper of Batakis is that it provides a constructive counterexample to a well-known paper by Edward Witten entitled Search for a Realistic Kaluza-Klein Theory, published in Nuclear Physics B (1981) 412-428, in which **Witten said**:

"...seven dimensions is in fact the minimum dimensionality of a manifold with $SU(3) \times SU(2) \times U(1)$ symmetry ... If, therefore, we wish to construct a theory in which $SU(3) \times SU(2) \times U(1)$ gauge fields arise as components of the gravitational field in more than four dimensions, we must have at least seven extra dimensions. ..."

It is sad that Witten's brilliant understanding of higher mathematics is accompanied by such a lack of physics intuition.

In a 1983-84 paper

Calculation of Gauge Couplings and Compact Circumferences from Self-Consistent Dimensional Reduction

by Candelas and Weinberg in Nuclear Physics B237 (1984) 397-441 (reprinted in a book

Modern Kaluza-Klein Theories

edited by Applequist, Chodos, and Freund (Addison-Wesley 1987), **Candelas and Weinberg** say:

"... we wish to show how fine-structure constants in general can be calculated in certain theories, in which the gauge fields arise from the metric of a higher-dimensional space. ... There are more general $(4+N)$ -dimensional models ... in which N dimensions form a compact manifold, and a massless gauge field appears in four dimensions for each Killing vector of this manifold. A general prescription has ... been given for calculating the various gauge couplings in such models in terms of the ratio of $2\pi (16\pi G)^{1/2}$ and various r.m.s. circumferences. ... In this paper we consider dynamical compactification ... The $(4+N)$ -dimensional space is again supposed to break up into a 4-dimensional Minkowski space and a curved compact N -dimensional manifold, with the curvature governed by Einstein's field equations. ... the energy-momentum tensor on the right-hand side on the right-hand side of these equations is ... supposed to arise ... from the one-loop fluctuations in various matter fields. ... the energy-momentum tensor is balanced by the curvature, and solutions are possible without mass parameters in the lagrangian, and with the scale of the compact manifold set by the gravitational constant ... For an N -dimensional compact manifold whose linear dimensions are of order ρ , the one-loop energy density of f light matter fields is of order $f \rho^{-(4-N)}$. The $(4+N)$ -dimensional gravitational constant \bar{G} is of order $G \rho^N$, and the Einstein tensor is of order $\rho^{(-$

2) . Hence ... $\rho^2 = G f$. The L-loop gravitational corrections to the one-loop matter energy density ... are less for $L \geq 2$ than the one-loop matter terms by a factor ... $(1/f)^{L-1}$. Also, the L-loop purely gravitational contributions to the energy density ... are less for $L \geq 1$ than the one-loop matter terms by a factor ... $(1/f)^L$ for f sufficiently large the scale of the compact manifold is of order $\sqrt{G f}$... For manifolds ... [including]... spheres, CPN, and manifolds of simple groups ... we can normalize the one free parameter ρ^2 in the metric ...

We now make the further assumptions that the matter fields are massless in $4+N$ dimensions ... When a $(4+N)$ -dimensional space breaks up into a 4-dimensional Minkowski space and a compact manifold, the perturbations of this metric appear in 4 dimensions in part as a set of massless fields: the Yang-Mills fields $A^\mu_a(x)$ and the gravitations field $g_{\mu\nu}(x)$.

... the classical Einstein-Hilbert action of pure gravity in $4+N$ dimensions yields in 4 dimensions an action .. where ... $F^{\mu\nu}_e$ are the Yang-Mills curls of those gauge fields A^μ_e that correspond to closed Killing curves of the compact manifold ... [if there were not many species of matter fields]... then G_0 could be identified as the Newton gravitational coupling constant G , and the normalization condition for the Yang-Mills fields would yield ... $g_e = (2\pi(16\pi G)^{1/2}/N_e s_e)$... [where]... s_e is the r.m.s. circumference of the manifold along these curves ... However ... assuming ... many species of matter fields ... radiative corrections generate induced ... terms ... [involving]... new dimensionless coefficients ... D_{N_e} and E_N ... we see that [for one-parameter manifolds ... of odd dimensionality]... the true Newton constant G is given by $(1/16\pi G) = (1/16\pi G_0) + (E_N/\rho^2)$...

Even a manifold that is stable against all deformations will become unstable if the temperature is raised above a critical value T_c ... This suggests that there is a dramatic phase transition at $T = T_c$, in which the compactified dimensions explode outward. One wonders ... whether our universe started with equal circumferences in all $3+N$ spatial directions, and became tightly contracted in N of those dimensions only when the temperature fell below the critical temperature T_c "

Note that the critical temperature for dimensional reduction is consistent with the [cosmology](#), [high-temperature physics](#), and [current-experiment-energy-level physics](#) of [the D4-D5-E6-E7-E8 VoDou Physics model](#).